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National Qualifications 2022 MODIFIED

Mark

X847/76/02

Mathematics Paper 2 Answer booklet

FRIDAY, 6 MAY

10:45 AM – 12:15 PM

X8477602

Fill in these boxes and read what is printed below.

Full name of centre

Town

Forename(s)

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Number of seat

Date of birth

Day Month Year

Scottish candidate number

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

1.(a)

$A(-1,-1) \quad B(2,-4) \quad C(7,3)$

$m_{AB} = \frac{-4-(-1)}{2-(-1)} = -1$

$m_{\perp C} = 1$

$y - 3 = 1(x-7)$

$y = x - 7 + 3$

$y = x - 4$

1.(b)

$Mid_{AC} \left(\frac{-1+7}{2}, \frac{-1+3}{2} \right)$

$Mid_{AC} \left(\frac{6}{2}, \frac{2}{2} \right)$

$Mid_{AC} (3,1)$

$m_{Median} = \frac{1-(-4)}{3-2} = 5$

$y - 1 = 5(x-3)$

$y = 5x - 15 + 1$

$y = 5x - 14$

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1.(c)

$y = 5x - 14$

$y = x - 4$

$x - 4 = 5x - 14$

$14 - 4 = 5x - x$

$\frac{10}{4} = \frac{4x}{4}$

$\frac{5}{2} = x = 2.5$

$y = 2.5 - 4$

$y = -1.5$

2.

$2x^2 - 8x + (4 - p) = 0$

$b^2 - 4ac > 0$

$(-8)^2 - 4(2)(4-p) > 0$

$64 - 8(4-p) > 0$

$64 - 32 + 8p > 0$

$32 + 8p > 0$

$8p > -32$

$p > -4$

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3.(a)

$4\sin x + 5\cos x$

$k\sin(x+a)$

$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$

$k\sin(x+a) = k\sin x\cos a + k\sin a\cos x$

$4\sin x + 5\cos x$

$k\sin x\cos a = 4\sin x$ $\quad$ $k\sin a\cos x = 5\cos x$

$k\cos a = 4$ $\quad$ $k\sin a = 5$

$\frac{k\sin a}{k\cos a} = \frac{5}{4}$

$\tan a = 1.25$

$a = \tan^{-1}(1.25)$

$a \approx 51.34°$, $a \approx 0.896$ rad

$k^2\sin^2 a + k^2\cos^2 a = 5^2 + 4^2$

$k^2(\sin^2 a + \cos^2 a) = 25 + 16$

$\sin^2 a + \cos^2 a = 1$

$k^2 \times 1 = 41$

$k^2 = 41$

$k = \sqrt{41}$

$\pi : 180$

$\frac{\pi}{180} \quad 1$

$\frac{\pi}{180} \times 51.34 \quad 51.34$

$4\sin x + 5\cos x = \sqrt{41}\sin(x + 0.896)$

3.(b)

$4\sin x + 5\cos x = 5.5$

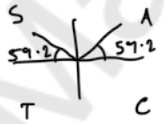
$\sqrt{41}\sin(x + 0.896) = 5.5$

$\sin(x + 0.896) = \frac{5.5}{\sqrt{41}}$

$x + 0.896 = \sin^{-1}\left(\frac{5.5}{\sqrt{41}}\right)$

$x = \sin^{-1}\left(\frac{5.5}{\sqrt{41}}\right) - 0.896$

$\sin^{-1}\left(\frac{5.5}{\sqrt{41}}\right) = 59.2°$, $180 - 59.2 = 120.8$



$59.2 \times \frac{\pi}{180} = 1.033$

$120.8 \times \frac{\pi}{180} = 2.108$

$x = 1.033 - 0.896$ $\quad$ $x = 2.108 - 0.896$

$x \approx 0.137$ $\quad$ $x \approx 1.212$

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4.(a)

$y = x^3 - 5x^2 + 2x + 8$

$$\int_{-1}^{2} (x^3 - 5x^2 + 2x + 8)\,dx$$

$$\left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x\right]_{-1}^{2}$$

$$\left[\frac{(2)^4}{4} - \frac{5(2)^3}{3} + (2)^2 + 8(2)\right] - \left[\frac{(-1)^4}{4} - \frac{5(-1)^3}{3} + (-1)^2 + 8(-1)\right]$$

$\frac{63}{4}$ square units

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4.(b)

$$\int_2^4 (x^3 - 5x^2 + 2x + 8)\,dx$$

$$\left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x\right]_2^4$$

$$\left[\frac{(4)^4}{4} - \frac{5(4)^3}{3} + (4)^2 + 8(4)\right] - \left[\frac{(2)^4}{4} - \frac{5(2)^3}{3} + (2)^2 + 8(2)\right]$$

$$-\frac{16}{3}$$

Total Shaded Area $= \frac{16}{3} + \frac{63}{4} = \frac{253}{12}$ square units

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5.(a) (i)

$f(x) = x^2 - 2 \qquad g(x) = 3x + 5$

$f(g(x)) = (3x+5)^2 - 2$

$9x^2 + 30x + 25 - 2$

$f(g(x)) = 9x^2 + 30x + 23$

5.(a) (ii)

$g(f(x)) = 3(x^2 - 2) + 5$

$3x^2 - 6 + 5$

$g(f(x)) = 3x^2 - 1$

5.(b)

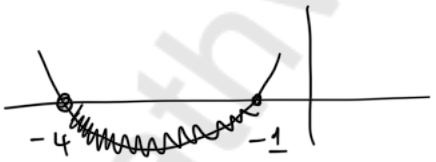
$9x^2 + 30x + 23 < 3x^2 - 1$

$6x^2 + 30x + 24 < 0$

$x^2 + 5x + 4 < 0$

$(x+1)(x+4) < 0$

critical values : $x = -1$, $x = -4$



$-4 < x < -1$

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6.

$$\frac{dy}{dx} = 1 - \frac{3}{x^2}$$

$$\frac{dy}{dx} = 1 - 3x^{-2}$$

$$y = \int (1 - 3x^{-2})\,dx$$

$$y = x - \frac{3x^{-1}}{-1} + c$$

$$y = x + \frac{3}{x} + c \qquad (3, 6)$$

$$6 = 3 + \frac{3}{3} + c$$

$$6 = 3 + 1 + c$$

$$c = 2$$

$$y = x + \frac{3}{x} + 2$$

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7.

$$y = kx^n$$

$$y = Mx + C$$

$$\log_5 y = m \log_5 x + 3$$

$$m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$$

$$\log_5 y = -2 \log_5 x + 3$$

$$\log_5 y = \log_5 x^{-2} + \log_5 125$$

$$\log_5 y = \log_5 125x^{-2}$$

$$5^{\log_5 y} = 5^{\log_5 125x^{-2}}$$

$$y = 125x^{-2}$$

$$y = kx^n$$

$$k = \underline{1}25 \qquad n = -2$$

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8.(a)

$A_T = x \times y = xy$

$150 = xy$

$y = \frac{150}{x}$

$A = xy - 1.5y - 1.5y - 1(x-3) - 1(x-3)$

$A = xy - 3y - 2x + 6$

$A = x\left(\frac{150}{x}\right) - 3\left(\frac{150}{x}\right) - 2x + 6$

$A = 150 - \frac{450}{x} - 2x + 6$

$A = 156 - 2x - \frac{450}{x}$

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8.(b)

$$A(x) = 156 - 2x - 450x^{-1}$$

$$A'(x) = -2 + 450x^{-2}$$

$$A''(x) = -900x^{-3}$$

Max occurs at $A'(x) = 0$

$$A''(15) = \frac{-900}{(15)^3} = -ve$$

15 is a local maximum

$$0 = -2 + \frac{450}{x^2}$$

$$2 = \frac{450}{x^2}$$

$$x^2 = \frac{450}{2}$$

$$x^2 = 225$$

$$x = \sqrt{225}$$

$$x = 15 \text{ m}$$

$$A_{max} = A(15) = 156 - 2(15) - \frac{450}{15}$$

$$A_{max} = \underline{96 \text{ m}^2}$$

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9.(a) $y = 3x + 7 \qquad x^2 + y^2 - 4x - 6y - 7 = 0$

$x^2 + (3x+7)^2 - 4x - 6(3x+7) - 7 = 0$

$x^2 + 9x^2 + 42x + 49 - 4x - 18x - 42 - 7 = 0$

$10x^2 + 20x = 0$

$10x(x+2) = 0$

$x = 0 \qquad x = -2$

$y = 7 \qquad y = 3(-2) + 7$

$y = 1$

$Q(0, 7) \quad P(-2, 1)$

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9.(b)

$x^2 + y^2 - 4x - 6y - 7 = 0$

$x^2 - 4x + y^2 - 6y = 7$

$(x-2)^2 + (y-3)^2 = 2^2 + 3^2 + 7$

$(x-2)^2 + (y-3)^2 = 20$

Centre $(2, 3)$

$M_{PQ}\left(\frac{0-2}{2}, \frac{7+1}{2}\right)$

$M_{PQ}\left(-\frac{1}{}, 4\right)$

$r^2 = (2-(-1))^2 + (3-4)^2$

$r^2 = 9 + \underline{1}$

$r^2 = \underline{1}0$

$(x-2)^2 + (y-3)^2 = \underline{1}0$ Equation of small circle

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10.(a)

$$P = 4.99087(42.5 - T)^{1.81}$$

At $T = 24.55$

$$P = 4.990087\ (42.5 - 24.55)^{1.81}$$

$$P \simeq 929$$

10.(b)

$$P = 0.1888807\ (D - 210)^{K}$$

$$850 = 0.1888807\ (600 - 210)^{K}$$

$$4501.951728 = 390^{K}$$

$$\ln\left(\frac{850}{0.1888807}\right) = \ln 390^{K}$$

$$\ln\left(\frac{850}{0.1888807}\right) = K \ln 390$$

$$K = \frac{\ln\left(\frac{850}{0.1888807}\right)}{\ln 390}$$

$$K \simeq \underline{1}.4\underline{1}$$

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