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National
Qualifications
2023 MODIFIED

Mark

X847/76/01 Mathematics Paper 1 (Non-calculator) Answer booklet

THURSDAY, 4 MAY
9:00 AM – 10:15 AM

X8477601

Fill in these boxes and read what is printed below.

Full name of centre

Town

Forename(s)

Surname

Number of seat

Date of birth
Day Month Year

Scottish candidate number

You must NOT use a calculator.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

X847760101

QUESTION NUMBER

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1.

$$y = x^{5/3} - \frac{10}{x^4}$$

$$y = x^{5/3} - 10x^{-4}$$

$$\frac{dy}{dx} = \frac{5}{3}x^{5/3-3/3} - 10 \times -4\, x^{-4-1}$$

$$\frac{dy}{dx} = \frac{5}{3}x^{2/3} + 40x^{-5}$$

2.

$P(-2, 6) \quad Q(10, 0)$

$$M\left(\frac{-2+10}{2}, \frac{6+0}{2}\right)$$

$$M(4, 3)$$

$$m_{PQ} = \frac{6-0}{-2-10} = \frac{6}{-12} = -\frac{1}{2}$$

$$m_{PQ_\perp} = 2$$

$$y - 3 = 2(x - 4)$$

$$y - 3 = 2x - 8$$

$$y = 2x - 5$$

QUESTION NUMBER

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3.

$$\log_5 x - \log_5 3 = 2$$

$$\log_5\left(\frac{x}{3}\right) = 2$$

$$5^{\log_5\left(\frac{x}{3}\right)} = 5^2$$

$$\frac{x}{3} = 25$$

$$x = 75$$

QUESTION NUMBER

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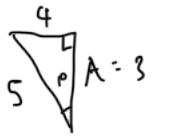
SOH CAH TOA

4.(a) (i)

$\cos\theta = \frac{A}{H}$

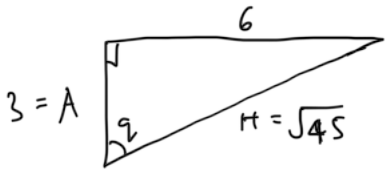


$A = \sqrt{5^2 - 4^2}$

$A = 3$

$\cos p = \frac{3}{5} = 0.6$

4.(a) (ii)



$H = \sqrt{6^2 + 3^2}$

$H = \sqrt{45}$

$\cos q = \frac{3}{\sqrt{45}}$

4.(b)

$\cos(p+q) = \cos p \cos q - \sin p \sin q$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin p = \frac{4}{5}$

$\sin q = \frac{6}{\sqrt{45}}$

$\cos(p+q) = \left(\frac{3}{5} \times \frac{3}{\sqrt{45}}\right) - \left(\frac{4}{5} \times \frac{6}{\sqrt{45}}\right)$

$\frac{9}{5\sqrt{45}} - \frac{24}{5\sqrt{45}}$

$\frac{-15}{5\sqrt{45}} \div 5 \div 5$

$\frac{-3}{\sqrt{45}}$

QUESTION NUMBER

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5.

$2x^2 + (3p-2)x + p = 0$

Equal roots ; $b^2 - 4ac = 0$

$a = 2 \quad b = 3p-2 \quad c = p$

$(3p-2)^2 - 4(2)(p) = 0$

$9p^2 - 12p + 4 - 8p = 0$

$9p^2 - 20p + 4 = 0$

36

$-1, -36$

$-2, -18$

$9p^2 - 18p - 2p + 4 = 0$

$9p(p-2) - 2(p-2) = 0$

$(9p-2)(p-2) = 0$

$9p - 2 = 0 \qquad p - 2 = 0$

$p = \frac{2}{9} \qquad p = 2$

6.

$\int 2x^5 - 6\sqrt{x}\, dx$

$\int 2x^5 - 6x^{1/2}\, dx$

$\frac{2x^6}{6} - \frac{6x^{3/2}}{3/2} + C$

$\frac{1}{3}x^6 - 4x^{3/2} + C$

QUESTION NUMBER

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7.(a)

$\log_2 5 + \log_2 \frac{1}{40}$

$\log_2 5 + \log_2 40^{-1}$

$\log_2 5 + (-\log_2 40)$

$\log_2 5 - \log_2 40$

$\log_2 \frac{5}{40}$

$\log_2 \frac{1}{8}$

$\log_2 8^{-1}$

$-\log_2 8$

$-\log_2 2^3$

$-3\log_2 2$

$-3 \times 1 = -3$

7.(b)

$0 < a < 1$

QUESTION NUMBER

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8.

$f(x) = x^3 + 3x^2 - 9x + 5$

$f'(x) = 3x^2 + 6x - 9$

$f'(x) = 0$ [stationary point occurs]

$0 = 3x^2 + 6x - 9$

$0 = x^2 + 2x - 3$

$0 = x^2 - x + 3x - 3$

$0 = x(x-1) + 3(x-1)$

$0 = (x+3)(x-1)$

Stationary Points at $x = 1$ and $x = -3$

$f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$

$f(1) = 0$

$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$

$= -27 + 27 + 27 + 5$

$f(-3) = 32$

Stationary Points: $(1, 0)$ $(-3, 32)$

$f''(x) = 6x + 6$

$f''(1) = 6 + 6 = 12$ (+ve)

hence local minimum at $(1, 0)$

$f''(-3) = 6(-3) + 6$

$-18 + 6 = -12$ (−ve)

hence local maximum at $(-3, 32)$

QUESTION NUMBER

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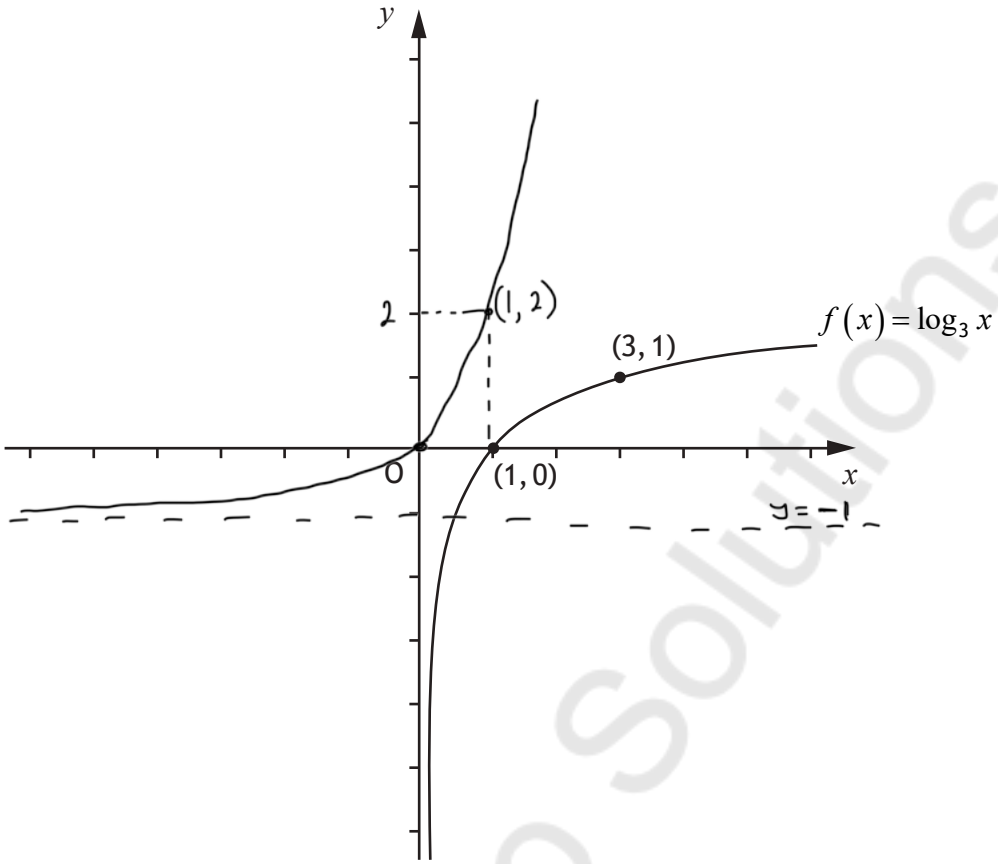
An additional diagram, if required, can be found on *page 13*.

9.



$y = \log_3 x$

$3^y = x$

$f^{-1}(x) = 3^x$

$y = 3^x - 1$

At $x = 0$

$y = 3^0 - 1$

$y = 1 - 1 = 0$

$(0, 0)$ ✓

At $x = 1$

$y = 3^1 - 1$

$y = 3 - 1 = 2$

$(1, 2)$ ✓

QUESTION NUMBER

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10.(a) $f(-5) = (-5)^4 + 3(-5)^3 - 7(-5)^2 + 9(-5) - 30$

$f(-5) = 625 - 375 - 175 - 45 - 30$

$f(-5) = 0$

Thus $x+5$ is a factor of $x^4 + 3x^3 - 7x^2 + 9x - 30$

10.(b)

$$x+5 \overline{)\, x^4 + 3x^3 - 7x^2 + 9x - 30}$$

$x^3 - 2x^2 + 3x - 6$

$-(x^4 + 5x^3)$

$-2x^3 - 7x^2$

$-(-2x^3 - 10x^2)$

$3x^2 + 9x$

$-(3x^2 + 15x)$

$-6x - 30$

$-6x - 30$

$0 \quad 0$

$(x+5)(x^3 - 2x^2 + 3x - 6) = 0$

Try 1 $(1)^3 - 2(1)^2 + 3(1) - 6$

$1 - 2 + 3 - 6$

$4 - 8 = -4$

Try 2 $(2)^3 - 2(2)^2 + 3(2) - 6$

$8 - 8 + 6 - 6 = 0$

$\therefore x - 2$ is a factor

$$x-2 \overline{)\, x^3 - 2x^2 + 3x - 6}$$

$x^2 + 3$

$-(x^3 - 2x^2)$

$0 + 3x - 6$

$-(3x - 6)$

$0 \quad 0$

$(x+5)(x-2)(x^2+3) = 0$

$x = -5 \quad x = 2 \quad x \in \mathbb{R}$

$x^2 + 3 = 0$

$x^2 = -3$

reject as x is imaginary

QUESTION NUMBER

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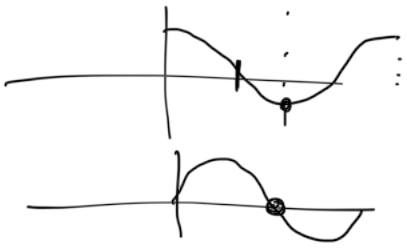
11.(a)

$$\int_{\frac{\pi}{2}}^{\pi} 5\sin x - 3\cos x \, dx$$



$$-5\cos x - 3\sin x \Big]_{\pi/2}^{\pi}$$

$$(-5\cos\pi - 3\sin\pi) - (-5\cos\tfrac{\pi}{2} - 3\sin\tfrac{\pi}{2})$$

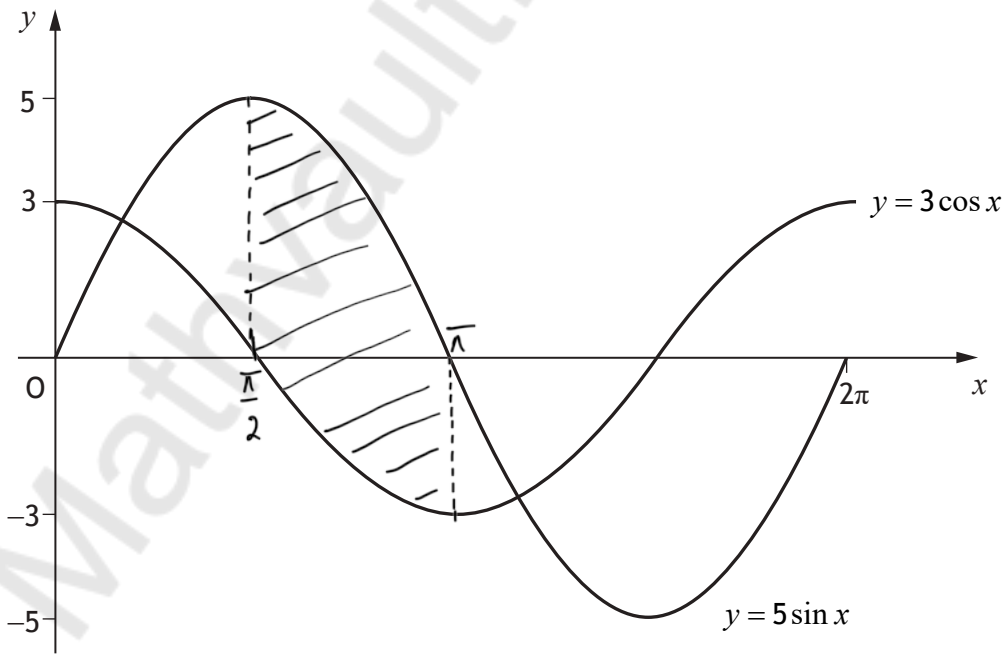
$$[-5(-1) - 3(0)] - [-5(0) - 3(1)]$$

$$5 - [-3]$$

$$5 + 3$$

$$8$$

11.(b) An additional diagram, if required, can be found on *page 14*.



QUESTION NUMBER

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12.

$$-2x^2 - 12x + 7$$

$$-2[x^2 + 6x] + 7$$

$$-2[(x+3)^2 - 9] + 7$$

$$-2(x+3)^2 + -2(-9) + 7$$

$$-2(x+3)^2 + 18 + 7$$

$$-2(x+3)^2 + 25$$

QUESTION NUMBER

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13.(a) (i)

$f(x) = 2\sin x$

$g(x) = 2x$

$g\left(\frac{\pi}{6}\right) = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$

$f\left(\frac{\pi}{3}\right) = 2\sin\frac{\pi}{3}$

$f\left(\frac{\pi}{3}\right) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

13.(a) (ii)

$f(g(x)) = 2\sin(2x)$

13.(b) (i)

$f(p) = \frac{1}{3}$

$2\sin p = \frac{1}{3}$

$\sin p = \frac{1}{6}$

13.(b) (ii)

$\sin p = \frac{1}{6}$

$\cos p = \frac{\sqrt{35}}{6}$

$\sqrt{6^2 - 1^2} = \sqrt{35}$

$f(g(p))$

$g(p) = 2p$

$f(2p) = 2\sin 2p$

$\sin 2x = 2\sin x \cos x$

$\sin 2p = 2\sin p \cos p$

$2\sin 2p = 4\sin p \cos p$

$f(2p) = 4 \times \frac{1}{6} \times \frac{\sqrt{35}}{6}$

$f(2p) = \frac{4\sqrt{35}}{36} = \frac{\sqrt{35}}{9}$

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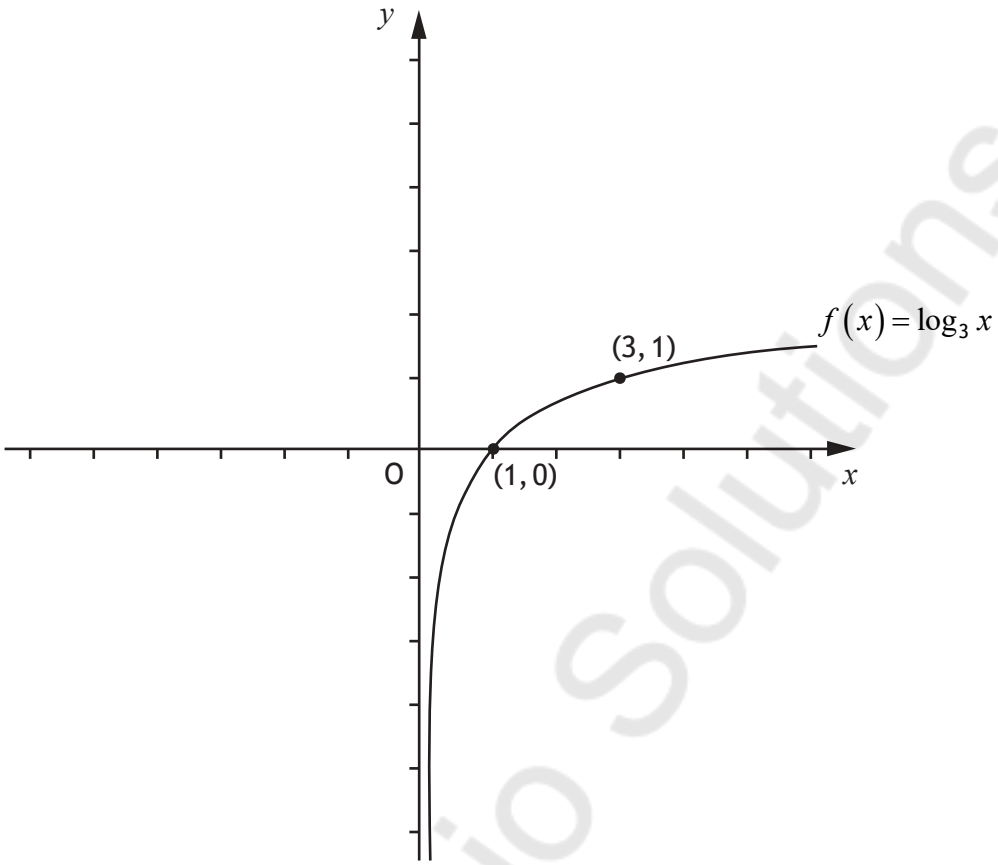
ADDITIONAL SPACE FOR ANSWERS

Additional diagram for question 9.



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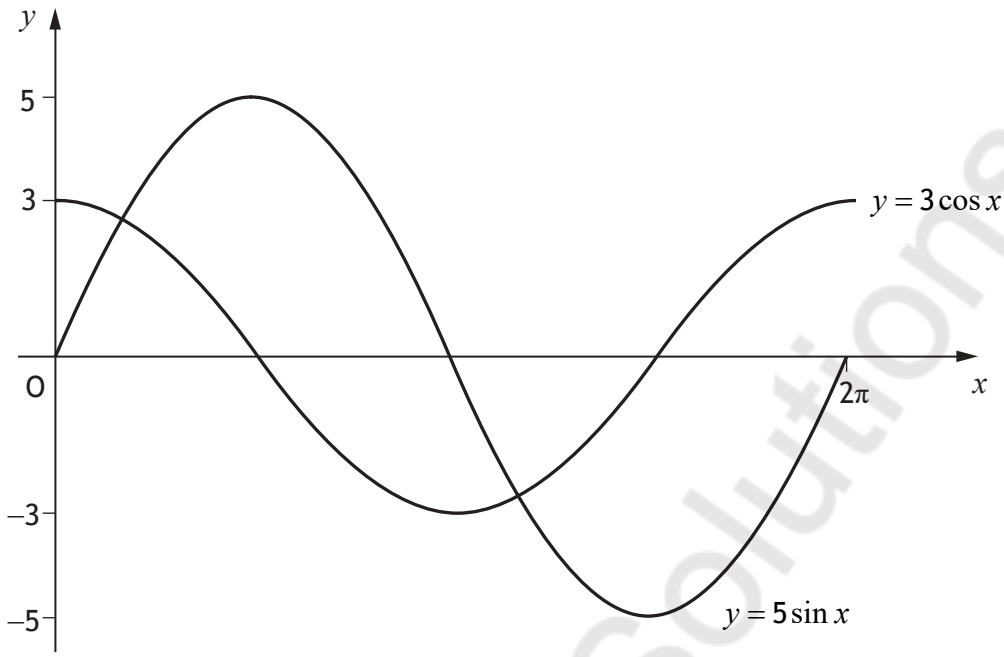
ADDITIONAL SPACE FOR ANSWERS

Additional diagram for question 11 (b).



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ADDITIONAL SPACE FOR ANSWERS

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For marker's use		
Question no	Marks/Grades	