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National Qualifications 2023 MODIFIED

Mark

X847/76/02

Mathematics Paper 2 Answer booklet

THURSDAY, 4 MAY

10:45 AM – 12:15 PM

X8477602

Fill in these boxes and read what is printed below.

Full name of centre

Town

Forename(s)

Surname

Number of seat

Date of birth

Day | Month | Year

Scottish candidate number

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

X8477602

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

1.(a) $P(5,-1)\quad Q(-2,8)\quad R(13,3)$

$$M_{QR} = \frac{8-3}{-2-13} = -\frac{1}{3}$$

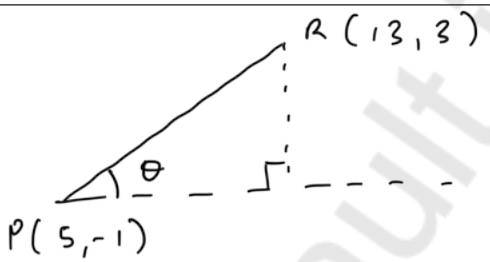
$$M_L = 3$$

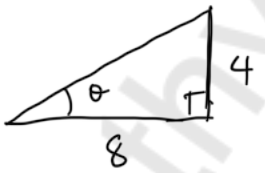
$$y-(-1) = 3(x-5)$$

$$y+1 = 3x-15$$

$$y = 3x-16$$

1.(b)





$$\tan\theta = \frac{4}{8}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\theta \simeq 26.6^\circ \text{ (1 d.p.)}$$

QUESTION NUMBER

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2.

$y = 2x^5 - 3x$

$\frac{dy}{dx} = 10x^4 - 3$

At $x = 1$

$\frac{dy}{dx} = 10(1)^4 - 3$

Gradient of tangent $= 7$

At $x = 1$

$y = 2(1)^5 - 3(1)$

$y = 2 - 3 = -1$

$(1, -1)$

$y - (-1) = 7(x - 1)$

$y + 1 = 7x - 7$

$y = 7x - 8$

3.

$\int 7\cos\left(4x + \frac{\pi}{3}\right) dx$

$u = 4x + \frac{\pi}{3}$

$\frac{du}{dx} = 4, \quad dx = \frac{du}{4}$

$7\int \cos u \frac{du}{4}$

$\frac{7}{4}\int \cos u \, du$

$\frac{7}{4}[\sin u] + C$

$\frac{7}{4}\sin u + C$

$\frac{7}{4}\sin\left(4x + \frac{\pi}{3}\right) + C$

QUESTION NUMBER

4.

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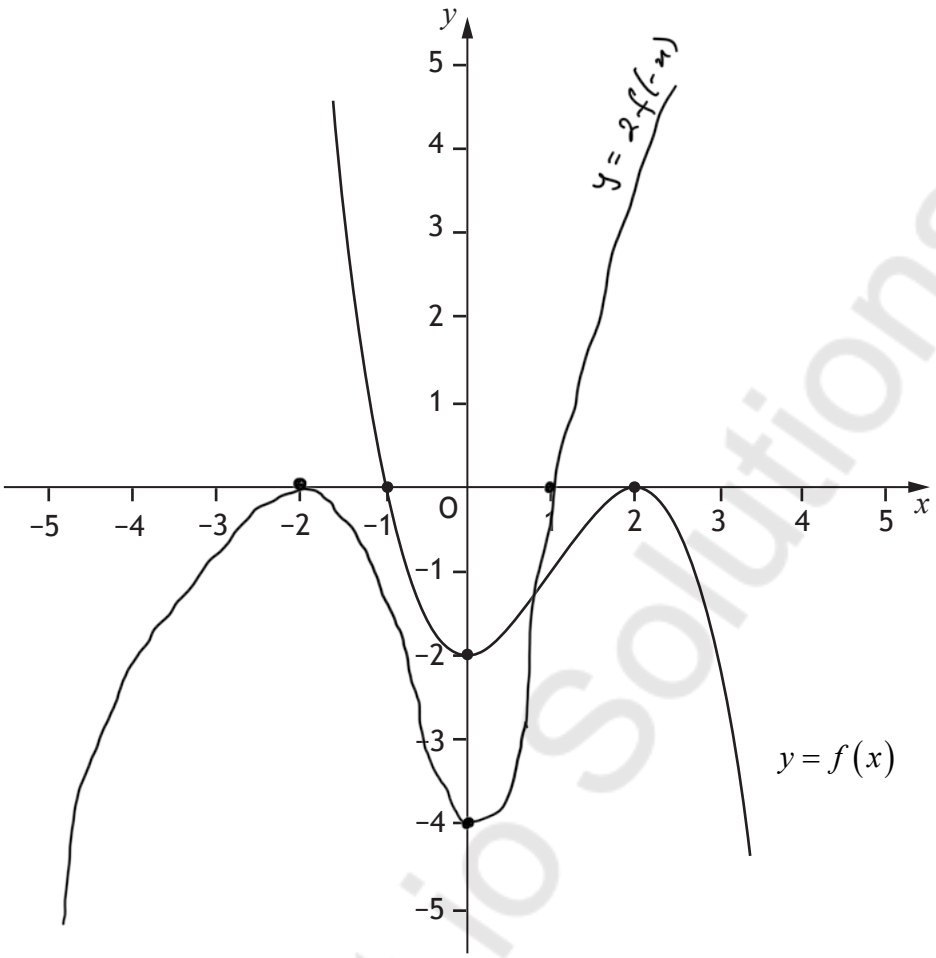
An additional diagram, if required, can be found on *page 16*.



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5.

$f(x) = (3-2x)^4$

$f'(x) = 4x - 2(3-2x)^3$

$f'(x) = -8(3-2x)^3$

$f'(4) = -8(3-2(4))^3$

$f'(4) = \underline{1}000$

6.

$f(x) = \frac{2}{x} + 3$

$x = \frac{2}{y} + 3$

$xy = 2 + 3y$

$xy - 3y = 2$

$y(x-3) = 2$

$y = \frac{2}{x-3}$

$f^{-1}(x) = \frac{2}{x-3}, \quad x \neq 3$

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7.

$\sin x + 2 = 3\cos 2x$

$\sin x + 2 = 3[\cos^2 x - \sin^2 x]$

$\sin x + 2 = 3\cos^2 x - 3\sin^2 x$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin x + 2 = 3[1 - \sin^2 x] - 3\sin^2 x$

$\sin x + 2 = 3 - 3\sin^2 x - 3\sin^2 x$

$6\sin^2 x + \sin x - 1 = 0$

$y = \sin x$

$6y^2 + y - 1 = 0$

$6x - 1 = -6$

$-1, 6$

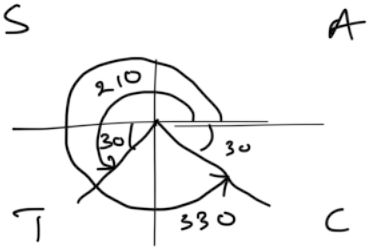
$-2, 3$

$6y^2 - 2y + 3y - 1 = 0$

$2y(3y - 1) + 1(3y - 1) = 0$

$(2y + 1)(3y - 1) = 0$

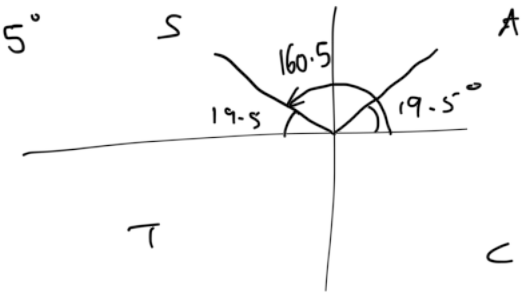
$y = -\frac{1}{2} \quad y = \frac{1}{3}$



$\sin x = -\frac{1}{2} \quad \sin x = \frac{1}{3}$

$x = \sin^{-1}(-0.5) \quad x = \sin^{-1}(\frac{1}{3})$

$x = 210°, 330° \quad x = 19.5°, 160.5°$



QUESTION NUMBER

8.

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$y = x^3 - 2x^2 - 4x + 1$

$y = x - 5$

$x - 5 = x^3 - 2x^2 - 4x + 1$

$0 = x^3 - 2x^2 - 5x + 6$

$$\int_{-2}^{1} (x^3 - 2x^2 - 5x + 6)\,dx$$

$$\left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x\right]_{-2}^{1}$$

$$\left[\frac{(1)^4}{4} - \frac{2(1)^3}{3} - \frac{5(1)^2}{2} + 6(1)\right] - \left[\frac{(-2)^4}{4} - \frac{2(-2)^3}{3} - \frac{5(-2)^2}{2} + 6(-2)\right]$$

$$\frac{37}{12} - \left(-\frac{38}{3}\right)$$

$$\frac{63}{4} \text{ units}^2$$

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9.(a)

$7\cos x - 3\sin x \qquad k\sin(x+a)$

$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$

$k\sin(x+a) = k\sin x\cos a + k\sin a\cos x$

$= -3\sin x + 7\cos x$

$\sqrt{58}\sin(x + 113.2)$

$k\sin x\cos a = -3\sin x \qquad k\sin a\cos x = 7\cos x$

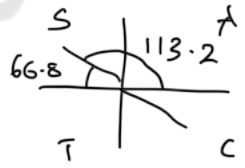
$k\cos a = -3 \qquad k\sin a = 7$

$k^2\sin^2 a + k^2\cos^2 a = (-3)^2 + (7)^2$

$k^2(\sin^2 a + \cos^2 a) = 9 + 49$

$k^2 = 58$

$k = \sqrt{58}$



$\frac{k\sin a}{k\cos a} = \frac{7}{-3}$

$\tan a = \frac{7}{-3}, \quad a = \tan^{-1}\left(\frac{7}{-3}\right), \quad a \approx 113.2^\circ$

9.(b) (i)

$14\cos x - 6\sin x = 2(7\cos x - 3\sin x)$

$14\cos x - 6\sin x = 2(\sqrt{58}\sin(x + 113.2))$

$2 \times \sqrt{58}$

max at $\sin(x + 113.2) = 1$

$2\sqrt{58}$

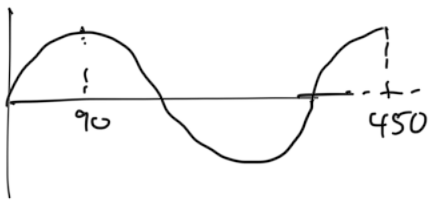
9.(b) (ii)

$\sin(x + 113.2) = 1$

$x + 113.2 = \sin^{-1}(1)$

$x + 113.2 = 450$

$x = 336.8^\circ$



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10. $f(x) = 2x^3 + 9x^2 - 24x + 6$

$f'(x) = 6x^2 + 18x - 24$

$f'(x) = 0$

$6x^2 + 18x - 24 = 0$

$x^2 + 3x - 4 = 0$

$(x - 1)(x + 4) = 0$

$x = 1 \quad x = -4$

$f''(x) = 12x + 18$

$f''(1) = 12 + 18 = 30$ (+ve)

$f''(-4) = 12(-4) + 18 = -30$ (-ve)

$x = 1$ local minimum

$x = -4$ local maximum

$f(-4) = 2(-4)^3 + 9(-4)^2 - 24(-4) + 6$

$f(-4) = 118$

$(-4, 118)$

$f(1) = 2(1)^3 + 9(1)^2 - 24(1) + 6$

$f(1) = -7$

$(1, -7)$

$-4 < x < 1$

QUESTION NUMBER

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11.(a) $C_1 : (x-4)^2 + (y+2)^2 = 37$ $\quad C_2 : x^2+y^2+2x-6y-7=0$

$C_1 (4, -2)$

$x^2+2x+y^2-6y = 7$

$(x+1)^2 - 1 + (y-3)^2 - 9 = 7$

$(x+1)^2 + (y-3)^2 = 17$

$C_2 (-1, 3)$

$D = \sqrt{[4-(-1)]^2 + [-2-3]^2}$

$= \sqrt{5^2 + (-5)^2}$

$D = 5\sqrt{2}$

11.(b) $C_1 : r = \sqrt{37}$ $\quad C_2 : r = \sqrt{17}$

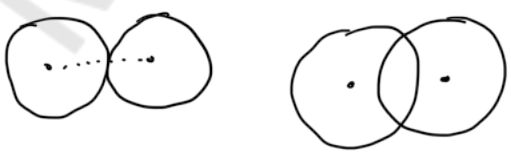
Distance between centres $= \sqrt{50} = 7.07$ (3 s.f.)

$\sqrt{37} - \sqrt{17} \approx 1.96$ (3 s.f.)

$\sqrt{37} + \sqrt{17} \approx 10.2$ (3 s.f.)

$1.96 < 7.07 < 10.2$

Thus C_1 and C_2 intersect at two distinct points



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12.

$$\frac{dy}{dx} = 8x^3 + 3$$

$$\int (8x^3 + 3)\,dx$$

$$y = \frac{8x^4}{4} + 3x + c \qquad \underset{x,\,y}{(-1,\,3)}$$

$$3 = 2(-1)^4 + 3(-1) + c$$

$$3 = 2 - 3 + c$$

$$c = 4$$

$$y = 2x^4 + 3x + 4$$

QUESTION NUMBER

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13.(a)

$C_t = 11e^{-0.0053t}$

At $t = 30$

$C_t = 11e^{-0.0053 \times 30}$

$C_t = 9.38$ mg/l (3 s.f.)

13.(b)

At $C_t = 0.66$ mg/l

$0.66 = 11e^{-0.0053t}$

$0.06 = e^{-0.0053t}$

$\ln 0.06 = \ln e^{-0.0053t}$

$\ln 0.06 = -0.0053t \ln e$

$\ln e = 1$

$\ln 0.06 = -0.0053t$

$\frac{\ln 0.06}{-0.0053} = t$

$530.832 \simeq t$

minutes

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14.(a) (i)

$(h \times 2x) + (2x \times 3x) + (3x \times h) + (2x \times h) + (3x \times h) = A$

$2xh + 6x^2 + 3xh + 2xh + 3xh = A$

$10xh + 6x^2 = A$

14.(a) (ii)

$10xh + 6x^2 = 7200$

$10xh = 7200 - 6x^2$

$h = \frac{7200 - 6x^2}{10x}$

$V = 3x \times 2x \times h$

$V = 6x^2 h$

$V = 6x^2 \times \left(\frac{7200 - 6x^2}{10x}\right)$

$V = 3x \times \left(\frac{7200 - 6x^2}{5}\right)$

$3x \times (1440 - 1.2x^2)$

$V = 4320x - \frac{18}{5}x^3$

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14.(b)

$V = 4320x - 3.6x^3$

$\frac{dV}{dx} = 4320 - 10.8x^2$

Max at $\frac{dV}{dx} = 0$

$10.8x^2 = 4320$

$x = \sqrt{\frac{4320}{10.8}}$

$x = \pm 20$

$x = 20$ cm maximizes the volume

$\frac{d^2V}{dx^2} = -21.6x$

At $x = 20$

$\frac{d^2V}{dx^2} = -21.6(20) = -432$ (-ve)

hence local maximum.

QUESTION NUMBER

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15. $C(0, y)$

$x + 3y = 17$

$3y = -x + 17$

$y = -\frac{1}{3}x + \frac{17}{3}$

$m_T = -\frac{1}{3}$

$m_r = 3$

$y - 5 = 3(x - 2)$

$y - 5 = 3x - 6$

$y = 3x - 1$

$y = 3(0) - 1$

$y = -1$

$C(0, -1)$

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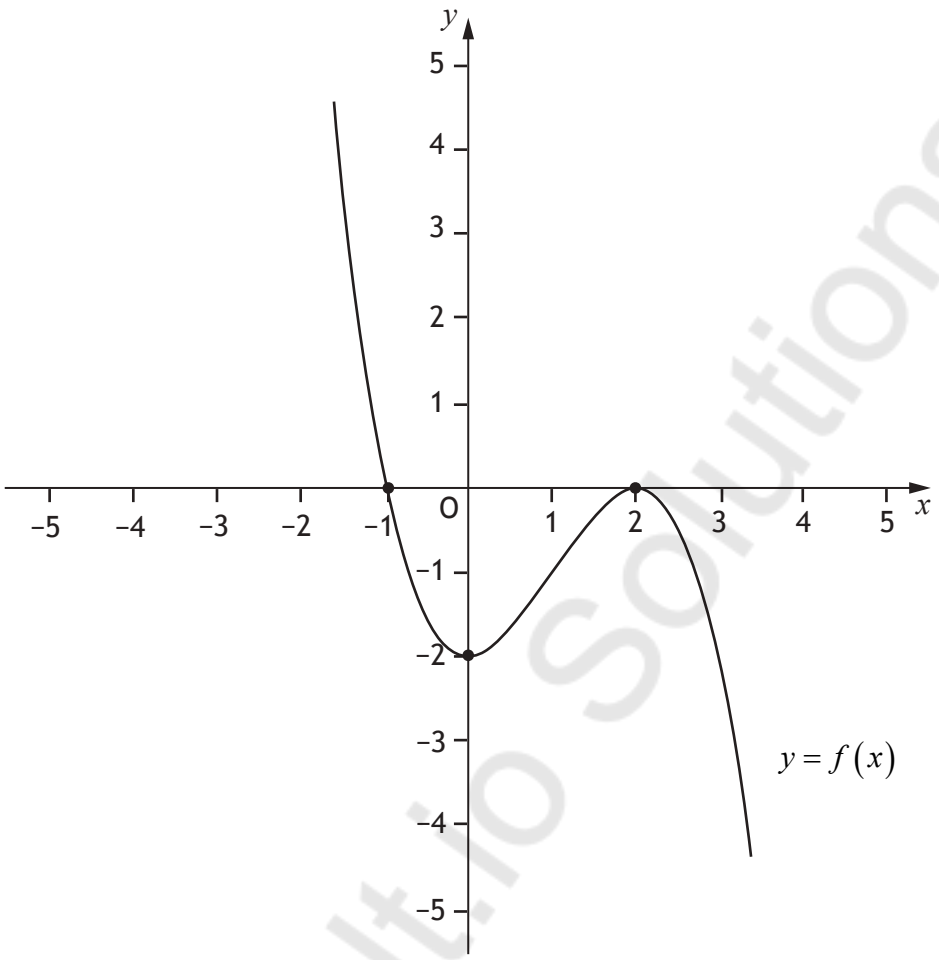
ADDITIONAL SPACE FOR ANSWERS

Additional diagram for question 4.



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Question no	Marks/Grades	