FOR OFFICIAL USE

National Qualifications 2024

Mark

X847/76/01 Mathematics Paper 1 (Non-calculator) Answer booklet

MONDAY, 13 MAY
9:00 AM – 10:15 AM

X8477601

Fill in these boxes and read what is printed below.

Full name of centre

Town

Forename(s)

Surname

Number of seat

Date of birth

Day | Month | Year

Scottish candidate number

You must NOT use a calculator.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

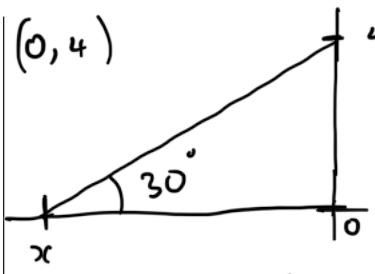
Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

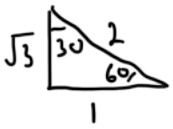
X8477601

QUESTION NUMBER

1.



$\tan 30 = \frac{4}{x}$



$\tan 30 = \frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{4}{x}$

$x = 4\sqrt{3}$

$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$(0, 4) \quad (-4\sqrt{3}, 0)$

$m = \frac{4 - 0}{0 - (-4\sqrt{3})}$

$m = \frac{4}{4\sqrt{3}}$

$m = \frac{1}{\sqrt{3}}$

$y - 4 = \frac{1}{\sqrt{3}}(x - 0)$

$y - 4 = \frac{1}{\sqrt{3}}x$

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2.(a)

$u_{n+1} = \frac{1}{5}u_n + 12$

$u_1 = 20$

$u_2 = \frac{1}{5} \times 20 + 12$

$u_2 = 4 + 12 = 16$

2.(b) (i)

$u_1 = 20$, $u_2 = 16$

The Sequence is a converging sequence as u_n gets Smaller.

2.(b) (ii)

$u_3 = \frac{1}{5} \times 16 + 12$

$u_3 = 3.2 + 12 = 15.2$

$u_4 = \frac{1}{5} \times 15.2 + 12$

$u_4 = 3.04 + 12$

$u_4 = 15.04$

$u_5 = \frac{1}{5} \times 15.04 + 12$

$u_5 = 3.008 + 12$

$u_5 = 15.008$

As $n \to \infty$

$u_n \longrightarrow 15$

QUESTION NUMBER

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3.

$y = (5x^2 + 3)^7$

$\frac{dy}{dx}$: $u = 5x^2 + 3$ $\quad y = u^7$

$\frac{du}{dx} = 10x$ $\quad \frac{dy}{du} = 7u^6$

$\frac{dy}{du} = 7(5x^2+3)^6$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$= 10x \times 7(5x^2+3)^6$

$\frac{dy}{dx} = 70x(5x^2+3)^6$

4.

$P(-6, 1, 2)$

$Q(-1, 11, -8)$

$\overrightarrow{OP} = \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}$

$\overrightarrow{OQ} = \begin{pmatrix} -1 \\ 11 \\ -8 \end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{pmatrix} -1 \\ 11 \\ -8 \end{pmatrix} - \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}$

$\overrightarrow{PQ} = \begin{pmatrix} -1 - (-6) \\ 11 - 1 \\ -8 - 2 \end{pmatrix}$

$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 10 \\ -10 \end{pmatrix}$

$\overrightarrow{PR} = \frac{2}{5}\overrightarrow{PQ}$ $\quad \overrightarrow{RQ} = \frac{3}{5}\overrightarrow{PQ}$

$\overrightarrow{OR} - \overrightarrow{OP} = \frac{2}{5}\overrightarrow{PQ}$

$\overrightarrow{OR} = \frac{2}{5}\overrightarrow{PQ} + \overrightarrow{OP}$

$\overrightarrow{OR} = \frac{2}{5}\begin{pmatrix} 5 \\ 10 \\ -10 \end{pmatrix} + \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}$

$\overrightarrow{OR} = \begin{pmatrix} \frac{2}{5} \times 5 \\ \frac{2}{5} \times 10 \\ \frac{2}{5} \times -10 \end{pmatrix} + \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}$

$\overrightarrow{OR} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} + \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2-6 \\ 4+1 \\ -4+2 \end{pmatrix}$

$\overrightarrow{OR} = \begin{pmatrix} -4 \\ 5 \\ -2 \end{pmatrix}$ $\quad R(-4, 5, -2)$

QUESTION NUMBER

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5.

$$h(x) = 2x^3 - 7$$

$$x = 2y^3 - 7$$

$$x + 7 = 2y^3$$

$$\frac{x+7}{2} = y^3$$

$$h^{-1}(x) = \left(\frac{x+7}{2}\right)^{1/3}$$

QUESTION NUMBER

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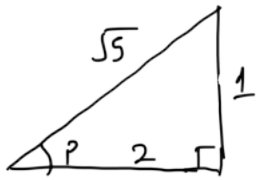
6.(a) (i)



$(\sqrt{5})^2 = 1^2 + A^2$

$5 - 1 = A^2$

$4 = A^2$

$2 = A$

$\sin p = \frac{1}{\sqrt{5}} \quad \cos p = \frac{2}{\sqrt{5}}$

$\sin 2x = 2\sin x \cos x$

$\sin 2p = 2\sin p \cos p$

$\sin 2p = 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$

$\sin 2p = \frac{4}{5}$

6.(a) (ii)

$\cos 2p = \cos^2 p - \sin^2 p$

$= \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

$= \frac{4}{5} - \frac{1}{5}$

$\cos 2p = \frac{3}{5}$

6.(b)

$\sin 4p$

$\sin 2(2p) = 2\sin 2p \cos 2p$

$= 2 \times \frac{4}{5} \times \frac{3}{5}$

$\sin 4p = \frac{24}{25}$

QUESTION NUMBER

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7. $x^2 - 14x + y^2 - 8y = -45$

$(x-7)^2 - 49 + (y-4)^2 - 16 = -45$

$(x-7)^2 + (y-4)^2 = 20$

$C(7,4)$

$y = 2x \rightarrow$ Tangent

$m_T = 2$ (Gradient of tangent)

$m_r = -\frac{1}{2}$ (Gradient of radius)

radius passes through centre

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}(x - 7)$

$2y - 8 = -x + 7$

$-2y + 15 = x$ [Equation of radius]

$y = 2(-2y + 15)$

$y = -4y + 30$

$5y = 30$

$y = 6$

$x = -2(6) + 15$

$x = -12 + 15$

$x = 3$

Point of Contact : $(3, 6)$

QUESTION NUMBER

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8.

$x^2 + (m-4)x + (2m-3) = 0$

No real roots $\therefore b^2 - 4ac < 0$

$b = m-4$, $a = 1$, $c = 2m-3$

$(m-4)^2 - 4(1)(2m-3) < 0$

$m^2 - 8m + 16 - 8m + 12 < 0$

$m^2 - 16m + 28 < 0$

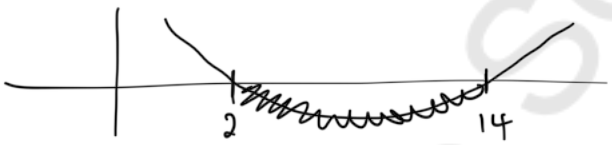
$m^2 - 2m - 14m + 28 < 0$

$m(m-2) - 14(m-2) < 0$

$(m-14)(m-2) < 0$

critical values: $m = 14$, $m = 2$

28
1, 28
2, 14
−2, −14



$2 < m < 14$

9.

$\log_a 5 + \log_a 80 - 2\log_a 10$

$\log_a 5 + \log_a 80 - \log_a 10^2$

$\log_a \left(\frac{5 \times 80}{10^2}\right)$

$\log_a \left(\frac{5 \times 80}{100}\right)$

$\log_a \left(\frac{80}{20}\right)$

$\log_a 4 \quad \therefore 12 = 4$

QUESTION NUMBER

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10.(a)

$x - 1 = 0$

$x = 1$

At $x = 1$, $2(1)^4 + 3(1)^3 - 4(1)^2 - 3(1) + 2$

$2 + 3 - 4 - 3 + 2$

0

Thus $x - 1$ is a factor of $2x^4 + 3x^3 - 4x^2 - 3x + 2$

10.(b)

$$x - 1 \;\overline{)\; 2x^4 + 3x^3 - 4x^2 - 3x + 2}$$

$2x^3 + 5x^2 + x - 2$

$-(2x^4 - 2x^3)$

$5x^3 - 4x^2$

$-(5x^3 - 5x^2)$

$x^2 - 3x$

$-(x^2 - x)$

$-2x + 2$

$-(-2x + 2)$

$0 \quad 0$

$2(2)^3 + 5(2)^2 + (2)$

$16 + 20 - 2 \cdot 2$

$(x - 1)(2x^3 + 5x^2 + x - 2)$

Via trial and error, $x + 2$ is also a factor

$\therefore$

$$x + 2 \;\overline{)\; 2x^3 + 5x^2 + x - 2}$$

$2x^2 + x - 1$

$-(2x^3 + 4x^2)$

$x^2 + x$

$-(x^2 + 2x)$

$-x - 2$

$-(-x - 2)$

$0 \quad 0$

$(x - 1)(x + 2)(2x^2 + x - 1)$

$(x - 1)(x + 2)(2x^2 + 2x - x - 1)$

$2x(x + 1) - 1(x + 1)$

$(x - 1)(x + 2)(2x - 1)(x + 1)$

QUESTION NUMBER

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11.(a) $\cos x + \sqrt{3}\sin x$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$k\cos(x - a) = k\cos x \cos a + k\sin x \sin a$

$\cos x + \sqrt{3}\sin x$

$k\cos x \cos a = \cos x$ $\quad$ $k\sin x \sin a = \sqrt{3}\sin x$

$k\cos a = 1$ $\quad$ $k\sin a = \sqrt{3}$

$k^2\cos^2 a = 1$ $\quad$ $k^2\sin^2 a = 3$

$k^2(\sin^2 a + \cos^2 a) = 1 + 3$ $\quad$ $\frac{k\sin a}{k\cos a} = \frac{\sqrt{3}}{1}$

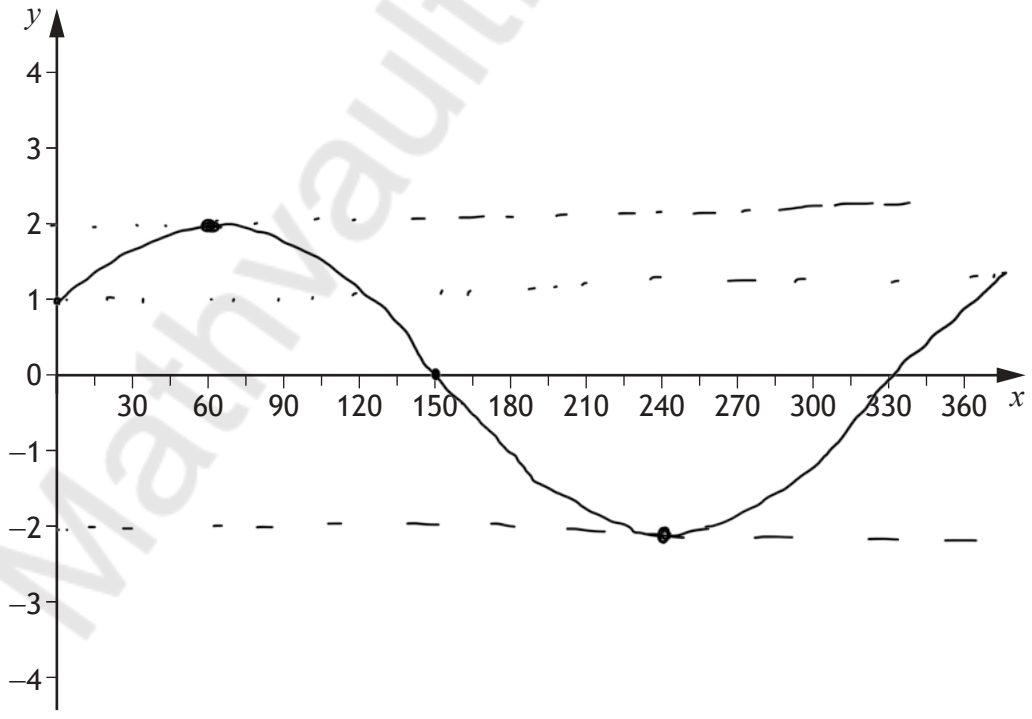
$\sin^2 a + \cos^2 a = 1$ $\quad$ $\tan a = \sqrt{3}$

$\therefore \quad k^2 = 4$ $\quad$ $a = 60°$

$k = 2$

$\cos x + \sqrt{3}\sin x = 2\cos(x - 60)$ ✓

11.(b) An additional diagram, if required, can be found on *page 12*.



QUESTION NUMBER

12.

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$$f(x) = 12x^{1/3}$$

$$f'(x) = \frac{1}{3} \times 12x^{1/3 - 1}$$

$$f'(x) = 4x^{-2/3}$$

$$f'(a) = \frac{1}{}$$

$$\frac{1}{} = 4(a)^{-2/3}$$

$$\frac{1}{} = \frac{4}{a^{2/3}}$$

$$(a^{2/3})^{1/2} = (4)^{1/2}$$

$$(a^{1/3})^3 = (2)^3$$

$$a = 8$$

QUESTION NUMBER

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13.(a) Midpoint of PQ : $\left(\frac{4+6}{2}, \frac{10+2}{2} \right)$

$M_{PQ} (5, 6)$

$m_{PQ} = \frac{10-2}{4-6} = \frac{8}{-2} = -4$

$m_{PQ\perp} = \frac{1}{4}$

Equation of $\perp$ bisector of PQ : $y - 6 = \frac{1}{4}(x - 5)$

$4y - 24 = x - 5$

$x - 4y + 19 = 0$

13.(b) $m_{QR} = 0$

$m_{QR\perp}$ = undefined $\therefore$ a vertical line

$M_{QR} \left(\frac{6+12}{2}, \frac{2+2}{2} \right)$

$M_{QR} (9, 2)$

$\therefore$ Perpendicular bisector of QR : $x = 9$

To find Centre. Equate $x = 9$ to $x + 19 - 4y = 0$

$y - 6 = \frac{1}{4}(9 - 5)$

$y - 6 = \frac{1}{4} \times 4$

$y - 6 = 1$

$y = 7$ Centre $(9, 7)$

Continued on Page 13

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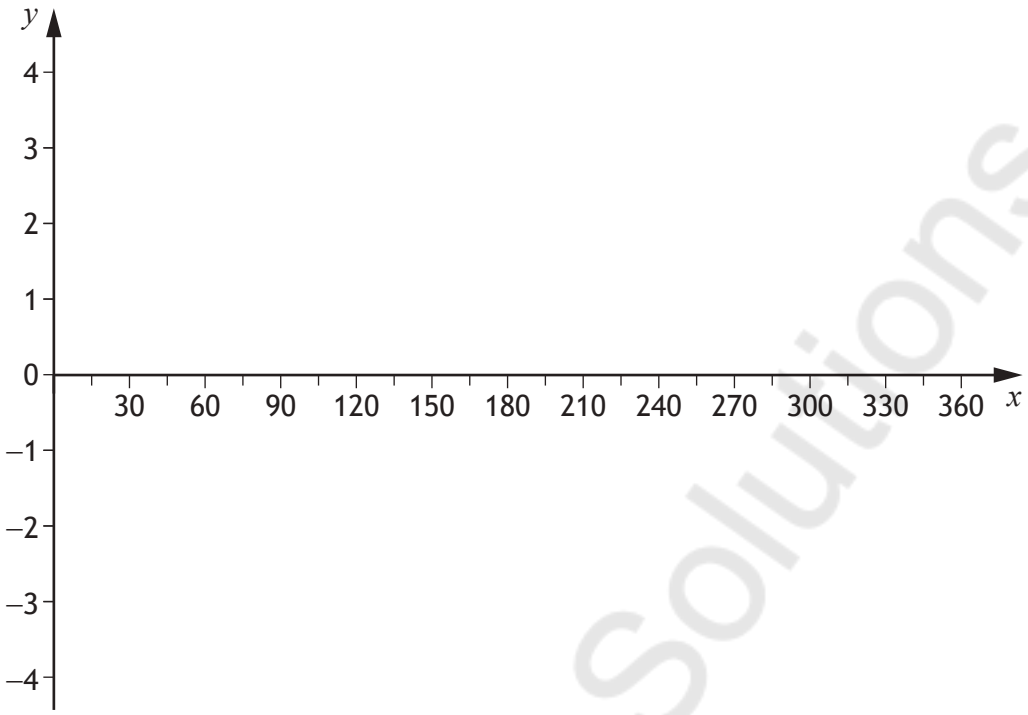
ADDITIONAL SPACE FOR ANSWERS

Additional diagram for question 11 (b).



Q13b Continued.

C(9, 7) R(12, 2)

$r^2 = (12-9)^2 + (2-7)^2$

$r^2 = 3^2 + (-5)^2$

$r^2 = 9 + 25$

$r^2 = 34$

Equation of circle: $(x-9)^2 + (y-7)^2 = 34$

ENTER NUMBER OF QUESTION

ADDITIONAL SPACE FOR ANSWERS

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Question	Marks