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National Qualifications 2024

Mark

X847/76/02

Mathematics Paper 2 Answer booklet

MONDAY, 13 MAY

10:45 AM – 12:15 PM

X8477602

Fill in these boxes and read what is printed below.

Full name of centre

Town

Forename(s)

Surname

Number of seat

Date of birth

Day | Month | Year

Scottish candidate number

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

X847760201

QUESTION NUMBER

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1.(a) $B(-1,-6)$ $A(-3,8)$ $C(11,0)$

$\text{Midpoint}_{AC} \left(\frac{-3+11}{2}, \frac{8+0}{2} \right)$

$\text{Midpoint}_{AC} \quad (4, 4)$

$m_{median} = \frac{4-(-6)}{4-(-1)} = 2$ Gradient of median

$y - 4 = 2(x - 4)$

$y = 2x - 8 + 4$

$y = 2x - 4$

1.(b) $B(-1,-6)$ $C(11,0)$

$m_{BC} = \frac{0-(-6)}{11-(-1)} = \frac{1}{2}$

$m_L = -2$

$y - 0 = -2(x - 11)$

$y = -2x + 22$

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1.(c)

Median line L

$y = 2x - 4$ $\quad$ $y = -2x + 22$

$2x - 4 = -2x + 22$

$4x = 26$

$x = \frac{26}{4} = \frac{13}{2} = 6.5$

$y = 2\left(\frac{13}{2}\right) - 4$

$y = 13 - 4$

$y = 9$

$(6.5, 9)$

2.

$y = \frac{8}{x^3}$ $\quad x > 0$

$y = 8x^{-3}$

$\frac{dy}{dx} = 8 \times -3 \times x^{-4}$

$\frac{dy}{dx} = -24x^{-4}$

$\frac{dy}{dx}$ at $x = 2$, $-24(2)^{-4}$

m_T at $x = 2 \Rightarrow -\frac{3}{2}$

$y = \frac{8}{(2)^3} = \frac{8}{8} = 1$

$(2, 1)$

$y - 1 = -\frac{3}{2}(x - 2)$

$2y - 2 = -3x + 6$

$3x + 2y = 8$

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3.(a)

$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ $\overrightarrow{OE} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ $\overrightarrow{OF} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$\overrightarrow{ED} = \overrightarrow{OD} - \overrightarrow{OE}$ $\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE}$

$\overrightarrow{ED} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ $\overrightarrow{EF} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$\overrightarrow{ED} = \begin{pmatrix} 2-1 \\ -3-1 \\ 4-(-2) \end{pmatrix}$ $\overrightarrow{EF} = \begin{pmatrix} 3-1 \\ 2-1 \\ 1-(-2) \end{pmatrix}$

$\overrightarrow{ED} = \begin{pmatrix} 1 \\ -4 \\ 6 \end{pmatrix}$ $\overrightarrow{EF} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

$\overrightarrow{ED} = i - 4j + 6k$ $\overrightarrow{EF} = 2i + j + 3k$

3.(b) (i)

$\overrightarrow{ED} \cdot \overrightarrow{EF} = (1 \times 2) + (-4 \times 1) + (6 \times 3)$

$\overrightarrow{ED} \cdot \overrightarrow{EF} = 2 - 4 + 18$

$\overrightarrow{ED} \cdot \overrightarrow{EF} = \underline{16}$

3.(b) (ii)

$\underbrace{a \cdot b}_{16} = |a||b| \cos\theta$

$|\overrightarrow{ED}| = \sqrt{(1)^2 + (-4)^2 + (6)^2} = \sqrt{53}$

$|\overrightarrow{EF}| = \sqrt{(2)^2 + (1)^2 + (3)^2} = \sqrt{14}$

$16 = \sqrt{53} \times \sqrt{14} \times \cos\theta$

$\cos\theta = \dfrac{16}{\sqrt{53}\sqrt{14}}$

$\theta = \cos^{-1}\left[\dfrac{16}{\sqrt{53} \times \sqrt{14}}\right]$

$\theta \approx \underline{54.028^\circ}$

QUESTION NUMBER

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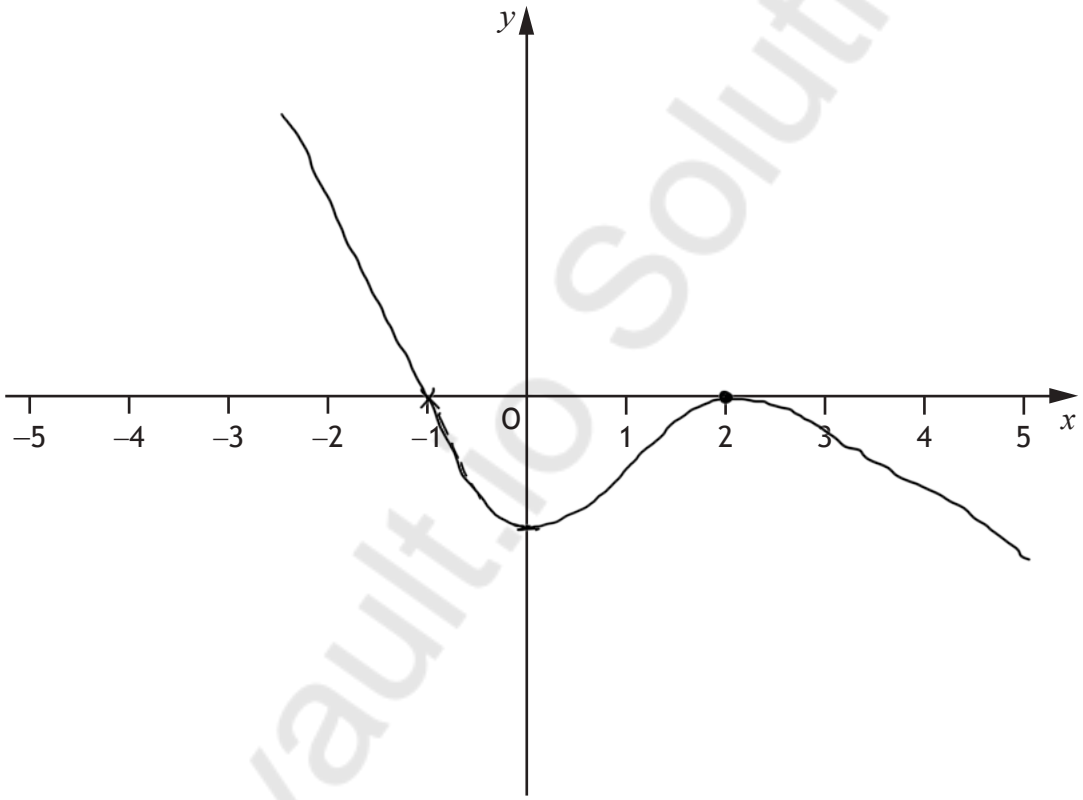
4.(a) $(-1, 3)$

$f(x-4)+2$

$(-1+4\ ,\ 3+2)$

$(3, 5)$

4.(b) An additional diagram, if required, can be found on *page 13*.



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5.

$$\int_0^{\pi/7} \sin 5x \, dx$$

let $u = 5x$

$u_2 = 5\left(\frac{\pi}{7}\right) = \frac{5\pi}{7}$

$u_1 = 5(0) = 0$

$$\int_0^{\frac{5\pi}{7}} \sin u \, dx$$

$u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5}$

$$\int_0^{\frac{5\pi}{7}} \sin u \, \frac{du}{5}$$

$$\frac{1}{5}\int_0^{\frac{5\pi}{7}} \sin u \, du$$

$$\frac{1}{5}\left[-\cos u\right]_0^{5\pi/7}$$

$$\frac{1}{5}\left[\left(-\cos\frac{5\pi}{7}\right) - \left(-\cos 0\right)\right]$$

0.3247 to 4 d.p.

6.

$y = ax^b$

$y = mx + c$

$\log_5 y = m \log_5 x - 2$

(4, 10) (0, -2)

$m = \frac{10-(-2)}{4-0}$

$m = 3$

$\log_5 y = 3\log_5 x - 2$

$\log_5 y = \log_5 x^3 - 2$

$\log_5 y - \log_5 x^3 = -2$

$\log_5 \frac{y}{x^3} = -2$

$5^{\log_5 \frac{y}{x^3}} = 5^{-2}$

$\frac{y}{x^3} = \frac{1}{25}$

$y = \frac{1}{25}x^3$

$y = ax^b$

$a = \frac{1}{25}$ $b = 3$

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7.

$$\text{Shaded Area} = \int_0^2 6 + 4x - 2x^2 \, dx - \int_0^2 x^3 - 6x^2 + 11x \, dx$$

$$\text{Shaded Area} = \left[6x + 2x^2 - \frac{2}{3}x^3 \right]_0^2 - \left[\frac{x^4}{4} - 2x^3 + \frac{11}{2}x^2 \right]_0^2$$

$$\left(6(2) + 2(2)^2 - \frac{2}{3}(2)^3 \right) - \left(\frac{(2)^4}{4} - 2(2)^3 + \frac{11}{2}(2)^2 \right)$$

$$\frac{44}{3} - 10$$

$$\text{Shaded Area} = \frac{14}{3} \text{ units}^2$$

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8.(a)

$f(x) = 2x^2 - 18 \qquad g(x) = x + 1$

$f(g(x)) = 2(x+1)^2 - 18$

$f(g(x)) = 2[x^2 + 2x + 1] - 18$

$2x^2 + 4x + 2 - 18$

$f(g(x)) = 2x^2 + 4x - 16$

8.(b)

$\frac{1}{f(g(x))} = \frac{1}{2x^2 + 4x - 16}$ } undefined when $2x^2 + 4x - 16 = 0$

$2x^2 + 4x - 16 = 0$

$x^2 + 2x - 8 = 0$

$(x - 2)(x + 4) = 0$

$x = 2 \qquad x = -4$

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9.(a)

$$y = \frac{x^3}{3} - x^2 - 3x + 1$$

$$\frac{dy}{dx} = 3 \times \frac{x^2}{3} - 2x - 3$$

when $\frac{dy}{dx} = 0$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \quad x = -1$$

$$y = \frac{(3)^3}{3} - (3)^2 - 3(3) + 1 = -8$$

$$y = \frac{(-1)^3}{3} - (-1)^2 - 3(-1) + 1 = \frac{8}{3}$$

$(3, -8)$ $\left(-1, \frac{8}{3}\right)$

9.(b)

$$\frac{d^2y}{dx^2} = 2x - 2$$

$x = 3$

$$\frac{d^2y}{dx^2} = 2(3) - 2 = 4 \text{ (+ve)}$$

hence minimum

$x = -1$

$$\frac{d^2y}{dx^2} = 2(-1) - 2 = -4 \text{ (-ve)}$$

hence maximum

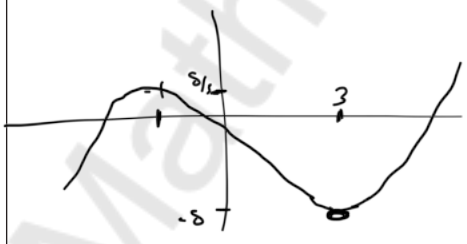
$y_{min} = -8$ for $-1 \leq x \leq 6$

At $x = 6$

$$y = \frac{(6)^3}{3} - (6)^2 - 3(6) + 1$$

$$y = 19$$

$y_{max} = 19$ for $-1 \leq x \leq 6$



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10.(a)

$x^2 + y^2 + 18x - 2y - 8 = 0$

$x^2 + 18x + y^2 - 2y = 8$

$(x + 9)^2 - 81 + (y - 1)^2 - 1 = 8$

$(x + 9)^2 + (y - 1)^2 = 90$

Centre $(-9, 1)$ $\quad r = \sqrt{90} = 3\sqrt{10}$

10.(b)

$C_1(-9, 1)$ $\quad C_2(-6, 0)$

$r_{C_2} = r_{C_1}$ - Distance between the two Centres

$\sqrt{[-6 - (-9)]^2 + [0 - 1]^2}$

$\sqrt{3^2 + (-1)^2}$

$\sqrt{10}$

$r_{C_2} = \sqrt{90} - \sqrt{10}$

Equation of Circle 2 : $(x + 6)^2 + y^2 = (\sqrt{90} - \sqrt{10})^2$

$(x + 6)^2 + y^2 = 40$

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11.(a) $N = 6.8e^{kt}$

When $t = 0$

$N = 6.8e^{k(0)}$

$N = 6.8$

At the end of 2020 there were 6.8 million electric vehicles worldwide.

11.(b) At $t = 10$, $N = 125$

$125 = 6.8e^{10k}$

$\frac{625}{34} = e^{10k}$

$\ln\left(\frac{625}{34}\right) = \ln e^{10k}$

$\ln 625 - \ln 34 = 10k \ln e$

$\ln 625 - \ln 34 = 10k$

$k = \frac{\ln 625 - \ln 34}{10}$

$k \simeq 0.29114$ to 5 d.p.

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12.

$2\sin 2x - \sin^2 x = 0$

$2[2\sin x \cos x] - \sin^2 x = 0$

$4\sin x \cos x - \sin^2 x = 0$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos x = \sqrt{1 - \sin^2 x}$

$4\sin x \sqrt{1-\sin^2 x} = \sin^2 x$

$4\sqrt{1-\sin^2 x} = \sin x$

$16(1-\sin^2 x) = \sin^2 x$

$16 - 16\sin^2 x = \sin^2 x$

$16 = 17\sin^2 x$

$\frac{16}{17} = \sin^2 x$

$\frac{4}{\sqrt{17}} = \sin x \qquad \frac{-4}{\sqrt{17}} = \sin x$

$x = \sin^{-1}\left(\frac{4}{\sqrt{17}}\right)$

$x = 75.96°, 180 - 75.96$

$x = 75.96°, 104.04°$

$x = \sin^{-1}\left(\frac{-4}{\sqrt{17}}\right)$

$x = 360 - 75.96$

$180 + 75.96$

$x = 284.04°, 255.96°$

13.

$(x+1)(x-5)(x-3)^2$

$(x^2 - 4x - 5)(x^2 - 6x + 9)$

$-5 \times 9 = -45$

$-\frac{1}{5} \times -45 = -9$

$f(x) = \frac{1}{5}(x-3)^2(x+1)(x-5)$

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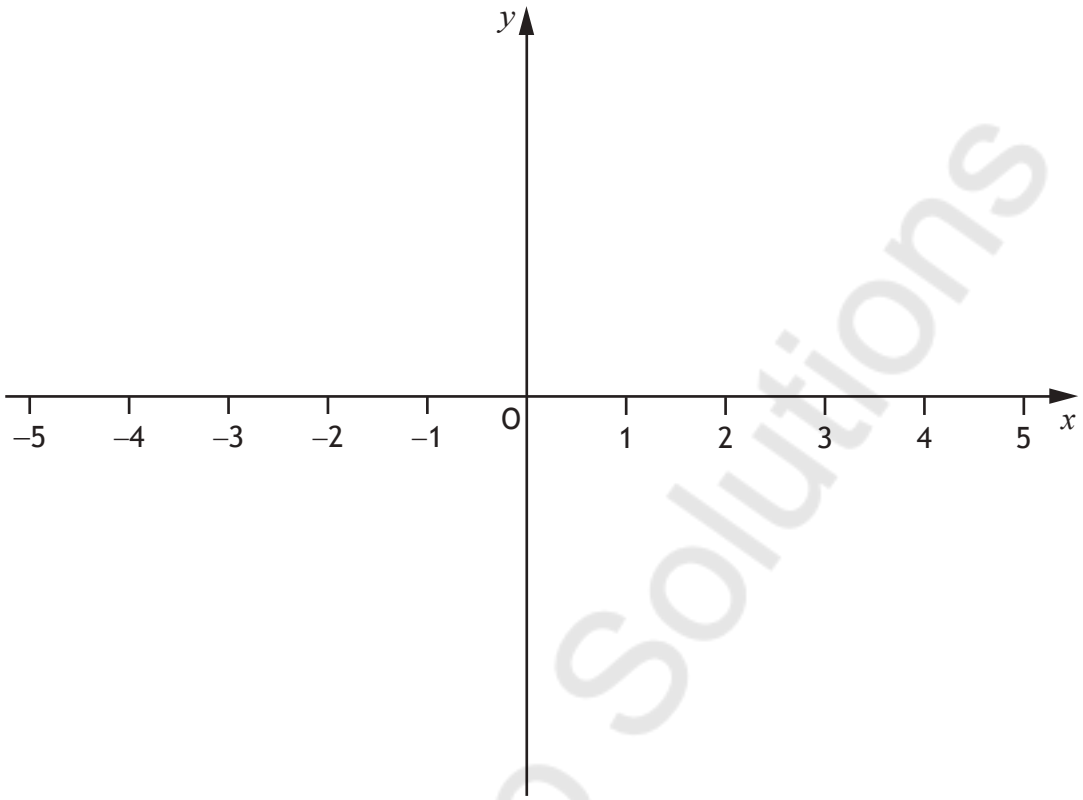
ADDITIONAL SPACE FOR ANSWERS

Additional diagram for question 4 (b).



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Question	Marks