H

National
Qualifications
2025

X847/76/11

Mathematics
Paper 1 (Non-calculator)

MONDAY, 12 MAY
9:00 AM – 10:15 AM

Total marks — 55

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

MARKS

Total marks — 55 marks

Attempt ALL questions

1. A curve has equation $y = x^3 - 2x^2 + 5$.

Find the equation of the tangent to this curve at the point where $x = 2$. **4**

$$\frac{dy}{dx} = 3x^2 - 4x$$

$$\frac{dy}{dx} = 3(2)^2 - 4(2)$$

$$m_T = 3(4) - 8$$

$$m_T = 12 - 8$$

$$m_T = 4$$

$$y - y_1 = m(x - x_1)$$

$$y = (2)^3 - 2(2)^2 + 5$$

$$y = 8 - 8 + 5$$

$$y = 5$$

$$\underset{x_1 \quad y_1}{(2, 5)}$$

$$y - 5 = 4(x - 2)$$

$$y - 5 = 4x - 8$$

$$\underline{\underline{y = 4x - 3}}$$

2. Find the equation of the perpendicular bisector of the line joining A (1, 4) and B (9, 10). **4**

$x_1 \; y_1 \qquad x_2 \; y_2$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \dfrac{10 - 4}{9 - 1}$

$m_{AB} = \dfrac{6}{8} = \dfrac{3}{4}$

$m_{AB\perp} = -\dfrac{4}{3}$

$M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$M\left(\dfrac{1+9}{2}, \dfrac{4+10}{2}\right)$

$M\left(\dfrac{10}{2}, \dfrac{14}{2}\right)$

$M\left(5, 7\right)$

$x_1 \; y_1$

$y - y_1 = m(x - x_1)$

$y - 7 = -\dfrac{4}{3}(x - 5)$

$y - 7 = -\dfrac{4}{3}x + \dfrac{20}{3}$

$y = -\dfrac{4}{3}x + \dfrac{20}{3} + \dfrac{7}{1}{}^{\times 3}_{\times 3}$

$y = -\dfrac{4}{3}x + \dfrac{20}{3} + \dfrac{21}{3}$

$y = -\dfrac{4}{3}x + \dfrac{41}{3}$

$3y = -4x + 41$

$3y + 4x = 41$

3. Find $\int\left(\frac{12}{x^2}+x^{\frac{1}{2}}\right)dx,\ x>0.$ 4

$$\int 12x^{-2} + x^{1/2}\, dx$$

$$\frac{12x^{-1}}{-1} + \frac{x^{3/2}}{3/2} + c$$

$$-12x^{-1} + \frac{2}{3}x^{3/2} + c$$

4. Evaluate $3\log_3 2+\log_3 \frac{1}{24}$. **3**

$\log_3 2^3 + \log_3 24^{-1}$

$\log_3 \left(2^3 \times 24^{-1}\right)$

$\log_3 \left(8 \times \frac{1}{24}\right)$

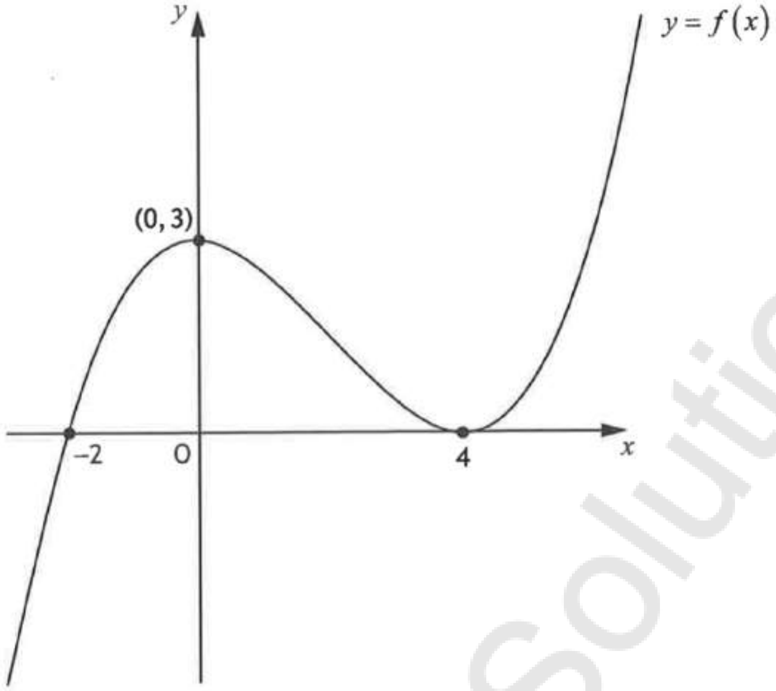
$\log_3 \frac{1}{3}$

$\log_3 3^{-1}$

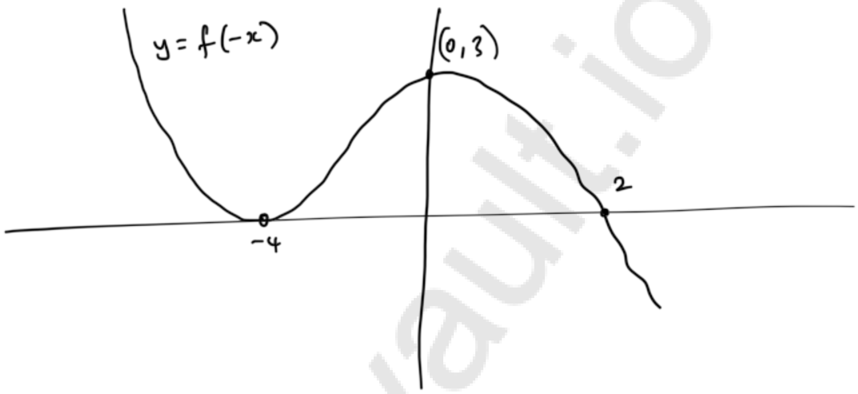
$-\log_3 3$

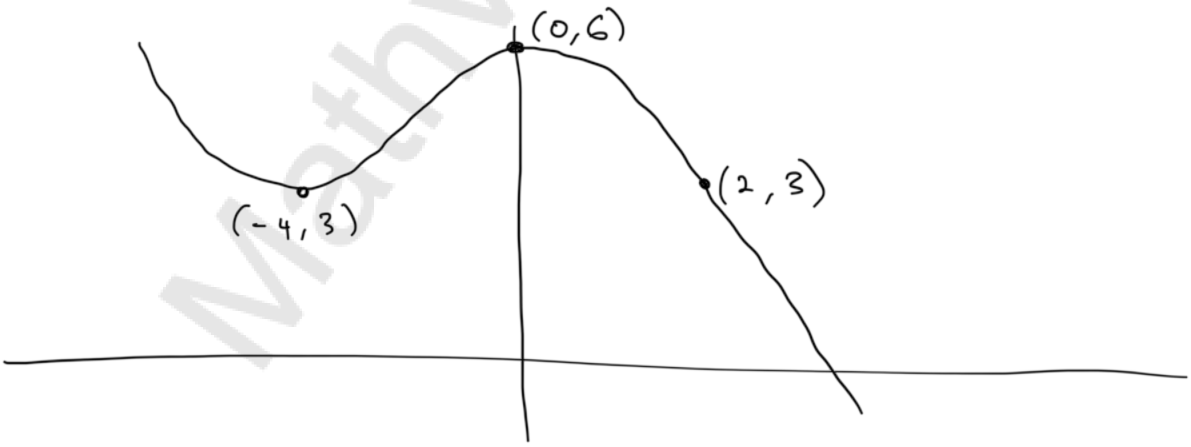
$\underline{\underline{-1}}$

5. The diagram shows the graph of $y = f(x)$, with stationary points at (0, 3) and (4, 0).

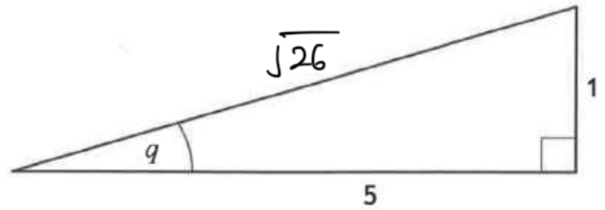


On the diagram in your answer booklet, sketch the graph of $y = f(-x) + 3$. **2**





6. The diagram shows a right-angled triangle with angle q.



(a) Determine the value of:

(i) $\sin 2q$ **3**

(ii) $\cos 2q$. **1**

a) i) $\sin 2q = 2\sin q \cos q$

$\sqrt{5^2 + 1^2} = \sqrt{26}$

$\sin q = \frac{1}{\sqrt{26}} \quad \cos q = \frac{5}{\sqrt{26}}$

$\sin 2q = 2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} = \frac{10}{26}$

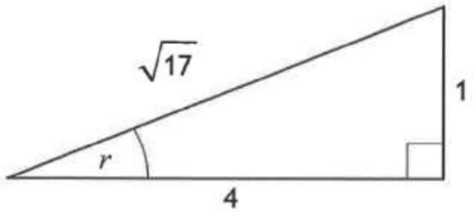
a) ii) $\cos 2q = \cos^2 q - \sin^2 q$

$\cos 2q = \left(\frac{5}{\sqrt{26}}\right)^2 - \left(\frac{1}{\sqrt{26}}\right)^2$

$= \frac{25}{26} - \frac{1}{26}$

$\cos 2q = \frac{24}{26}$

A second right-angled triangle has angle r as shown.



(b) Find the value of $\sin(2q-r)$. **3**

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\sin(2q-r) = \sin 2q \cos r - \sin r \cos 2q$$

$$\sin r = \frac{1}{\sqrt{17}} \qquad \cos r = \frac{4}{\sqrt{17}}$$

$$\sin(2q-r) = \left(\frac{10}{26} \times \frac{4}{\sqrt{17}}\right) - \left(\frac{1}{\sqrt{17}} \times \frac{24}{26}\right)$$

$$\left(\frac{5}{13} \times \frac{4}{\sqrt{17}}\right) - \left(\frac{1}{\sqrt{17}} \times \frac{12}{13}\right)$$

$$\frac{20}{13\sqrt{17}} - \frac{12}{13\sqrt{17}}$$

$$\sin(2q-r) = \frac{8}{13\sqrt{17}}$$

7. (a) Show that $(x+3)$ is a factor of $5x^3+16x^2-x-12$. **2**

(b) Hence, or otherwise, solve $5x^3+16x^2-x-12=0$. **3**

(a) $f(-3) = 5(-3)^3+16(-3)^2-(-3)-12$

$5(-27)+16(9)+3-12$

$-135+144-9$

$9-9$

$f(-3)=0$ $\therefore$ $x+3$ is a factor

$$\begin{array}{r} 27 \\ \times\ 5 \\ \hline 135 \end{array} \qquad \begin{array}{r} 16 \\ \times\ 9 \\ \hline 144 \end{array} \qquad \begin{array}{r} 144 \\ -135 \\ \hline 9 \end{array}$$

(b)

$$\begin{array}{r|l} & 5x^2 + x - 4 \\ \hline x+3 & 5x^3 + 16x^2 - x - 12 \\ & -(5x^3+15x^2) \\ \hline & x^2 - x \\ & -(x^2+3x) \\ \hline & -4x - 12 \\ & -4x - 12 \\ \hline & 0 \quad 0 \end{array}$$

-20

$+5, -4$

$(5x^2+x-4)(x+3)=0$

$(5x^2+5x-4x-4)(x+3)=0$

$[5x(x+1)-4(x+1)](x+3)=0$

$(5x-4)(x+1)(x+3)=0$

$x=\frac{4}{5}$ $x=-1$ $x=-3$

8. Given that $\log_a 75 = 2 + \log_a 3$, $\underline{a > 0}$, find the value of a. **3**

$$\log_a 75 = 2 + \log_a 3$$

$$\log_a 75 = 2\log_a a + \log_a 3$$

$$\log_a 75 = \log_a a^2 + \log_a 3$$

$$\log_a 75 = \log_a 3a^2$$

$$a^{\log_a 75} = a^{\log_a 3a^2}$$

$$\frac{75}{3} = \frac{3a^2}{3}$$

$$25 = a^2$$

$$\underline{\underline{a = 5}} \qquad \underset{\text{reject}}{a = -5}$$

9. Find the coordinates of the points of intersection of the line with equation $y = x+1$ and the circle with equation $x^2 + y^2 - 2x + 6y - 15 = 0$. **4**

$$x^2 + (x+1)^2 - 2x + 6(x+1) - 15 = 0$$

$$x^2 + x^2 + 2x + 1 - 2x + 6x + 6 - 15 = 0$$

-16

$+8, -2$

$$2x^2 + 6x - 8 = 0$$

$$2x^2 - 2x + 8x - 8 = 0$$

$$2x(x-1) + 8(x-1) = 0$$

$$(2x+8)(x-1) = 0$$

$$x = -4 \qquad x = 1$$

$$y = -4+1 \qquad y = 1+1$$

$$y = -3 \qquad y = 2$$

$$(-4, -3) \qquad (1, 2)$$

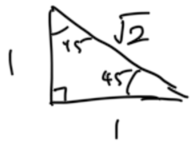
10. The vectors **u** and **v** are such that:

- $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$
- $\mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ k \end{pmatrix}$
- the angle between **u** and **v** is 45°.

Find the value of k, where $k > 0$. **5**

$u \cdot v = |\vec{u}| \times |\vec{v}| \times \cos\theta$

$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ k \end{pmatrix} = \sqrt{1^2+1^2+0^2} \times \sqrt{1^2+3^2+k^2} \times \cos 45$



$\cos 45 = \frac{1}{\sqrt{2}}$

$(1\times1) + (1\times3) + (0\times k) = \sqrt{2} \times \sqrt{10+k^2} \times \frac{1}{\sqrt{2}}$

$4 = \sqrt{10+k^2}$

$16 = 10 + k^2$

$6 = k^2$

$\underline{k = \sqrt{6}}$ $\quad k = -\sqrt{6}$

reject

11. The equation $9x^2 + 3kx + k = 0$ has two real and distinct roots.

Determine the range of values for k.

Justify your answer. 4

$b^2 - 4ac > 0$ 2 real roots distinct

$b = 3k$, $a = 9$, $c = k$

$(3k)^2 - 4(9)(k) > 0$

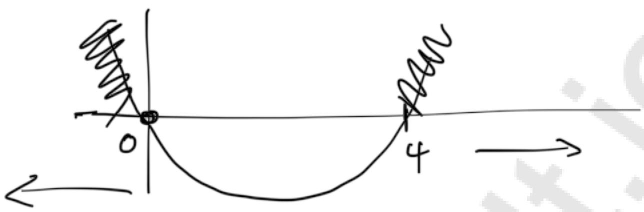
$9k^2 - 36k > 0$

$9(k^2 - 4k) > 0$

$k^2 - 4k > 0$

$k(k-4) > 0$

Critical values : $k = 0$ $k = 4$



$k < 0$ $k > 4$

12. Given that:

- $\frac{dy}{dx} = 6\cos x + 8\sin 2x$, and
- $y = 4$ when $x = \frac{\pi}{6}$,

express y in terms of x. **4**

$$y = \int 6\cos x + 8\sin 2x \, dx$$

$$6\sin x - \frac{8\cos 2x}{2} + C$$

$$y = 6\sin x - 4\cos 2x + C$$

$$4 = 6\sin\frac{\pi}{6} - 4\cos\frac{\pi}{3} + C$$

$\frac{180}{6} = 30$

$\sin 30 = \frac{1}{2}$

$\frac{180}{3} = 60$

$$4 = 6\left(\tfrac{1}{2}\right) - 4\left(\tfrac{1}{2}\right) + C$$

$$4 = 3 - 2 + C$$

$$4 = 1 + C$$

$$C = 3$$

$$\therefore \underline{\underline{y = 6\sin x - 4\cos 2x + 3}}$$

13. A function, f, is defined on the set of real numbers.

The derivative of f is $f'(x) = (x+5)(2-x)$.

(a) Find the x-coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **3**

$f'(x) = (x+5)(2-x)$

$2x - x^2 + 10 - 5x$

$f'(x) = -x^2 - 3x + 10$

$f(x) = \int -x^2 - 3x + 10 \, dx$

$f(x) = \frac{-x^3}{3} - \frac{3x^2}{2} + 10x + C$

$f'(x) = 0$, $(x+5)(2-x) = 0$

$x = -5 \quad x = 2$

$f''(x) = -2x - 3$

$f''(-5) = -2(-5) - 3 = 10 - 3 = 7$ (+ve) hence local minimum

$f''(2) = -2(2) - 3 = -4 - 3 = -7$ (-ve) hence local maximum

It is known that:

- f is a cubic function
- $f(0) < 0$
- the equation $f(x) = 0$ has exactly one solution. The solution lies between −10 and 10.

(b) Draw a sketch of a possible graph of $y = f(x)$ on the diagram in your answer booklet. 3

$$f'(x) = (x+5)(2-x)$$

$$f'(x) = -x^2 - 3x + 10$$

$$f(x) = \int -x^2 - 3x + 10 \, dx$$

$$f(x) = \frac{-x^3}{3} - \frac{3x^2}{2} + 10x + C$$

$f(0) < 0$

$C < 0$ ∴ −ve y-intercept