H

National Qualifications 2025

X847/76/12

Mathematics Paper 2

MONDAY, 12 MAY

10:45 AM – 12:15 PM

Total marks — 65

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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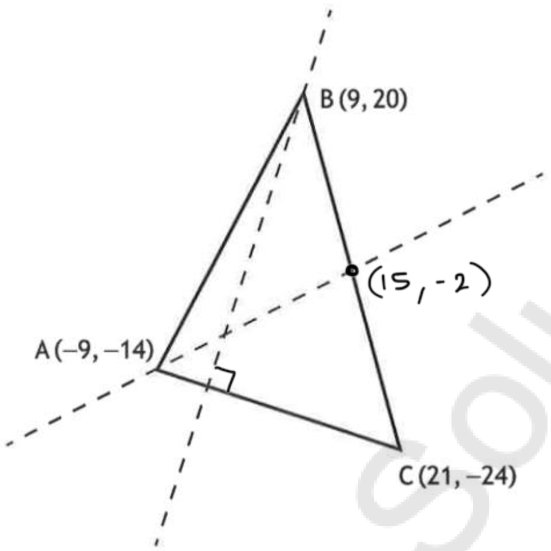
A/HTP

MARKS

Total marks — 65

Attempt ALL questions

1. Triangle ABC has vertices A (−9, −14), B (9, 20) and C (21, −24).



(a) Find the equation of the altitude through B. 3

(b) Find the equation of the median through A. 3

(c) Determine the point of intersection of the altitude through B and the median through A. 2

(a) $m_{AC} = \frac{-24-(-14)}{21-(-9)} = -\frac{1}{3}$

$m_{\perp B} = 3$

$y - 20 = 3(x - 9)$

$y - 20 = 3x - 27$

$y = 3x - 7$ -> Equation of the altitude through B.

(b) $\text{Midpoint}_{BC} \left(\frac{9+21}{2}, \frac{20+(-24)}{2}\right)$

$\left(\frac{30}{2}, \frac{20-24}{2}\right)$

$\left(15, \frac{-4}{2}\right)$

$\text{Midpoint}_{BC}\ (15, -2)$

Mid $(15, -2)$ $\quad$ $A(-9, -14)$

$$M_{\text{bisector}} = \frac{-14-(-2)}{-9-15} = \frac{1}{2}$$

$$y-(-14) = \frac{1}{2}(x-(-9))$$

$$y+14 = \frac{1}{2}x + \frac{9}{2}$$

$$y = \frac{1}{2}x - \frac{19}{2} \longrightarrow \text{Equation of median through } A$$

(c)

$$\frac{1}{2}x - \frac{19}{2} = 3x - 7$$

$$x - 19 = 6x - 14$$

$$-19+14 = 6x - x$$

$$-5 = 5x$$

$$x = -1$$

$$y = 3x - 7$$

$$y = 3(-1) - 7$$

$$y = -10$$

Point where the median through A and the altitude through B meet is $(-1, -10)$

2. Express $2x^2+16x+5$ in the form $p(x+q)^2+r$. **3**

$$2(x^2+8x)+5$$

$$2\left[(x+4)^2-(4)^2\right]+5$$

$$2\left[(x+4)^2-16\right]+5$$

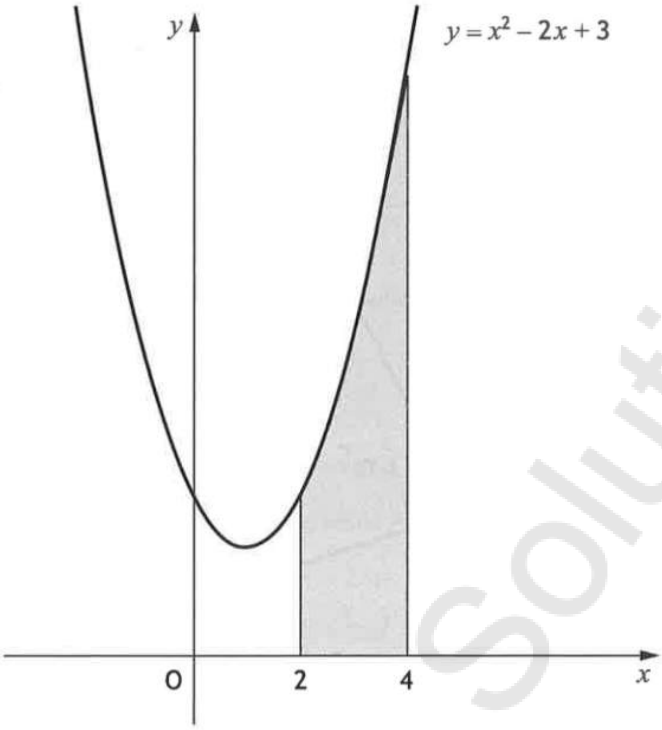
$$2(x+4)^2-32+5$$

$$2(x+4)^2-27$$

$p=2$, $q=4$, $r=-27$

MARKS

3. The diagram shows the graph of $y = x^2 - 2x + 3$.



Calculate the shaded area. **4**

$$\text{Area} = \int_2^4 (x^2 - 2x + 3)\,dx$$

$$\text{Area} = \left[\frac{x^3}{3} - x^2 + 3x\right]_2^4$$

$$\text{Area} = \left[\frac{(4)^3}{3} - (4)^2 + 3(4)\right] - \left[\frac{(2)^3}{3} - (2)^2 + 3(2)\right]$$

$$\text{Area} = \frac{52}{3} - \frac{14}{3}$$

$$\text{Area} = \frac{38}{3} \text{ square units}$$

4. A function, g, is defined by $g(x) = (x-4)^3$, where $x \in \mathbb{R}$.

Find the inverse function, $g^{-1}(x)$. **3**

$$x = (y-4)^3$$

$$x^{1/3} = y - 4$$

$$x^{1/3} + 4 = y$$

$$g^{-1}(x) = x^{1/3} + 4$$

5. (a) Show that the points A $(-3, 2, -1)$, B $(6, -1, 5)$ and C $(12, -3, 9)$ are collinear. **3**

(b) State the ratio in which B divides AC. **1**

(a) $\vec{OA} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ $\quad \vec{OB} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$ $\quad \vec{OC} = \begin{pmatrix} 12 \\ -3 \\ 9 \end{pmatrix}$

$K \times \vec{AB} = \vec{BC}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{BC} = \vec{OC} - \vec{OB}$

$\vec{AB} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 6-(-3) \\ -1-2 \\ 5-(-1) \end{pmatrix} = \vec{AB} = \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix}$

$\vec{BC} = \begin{pmatrix} 12 \\ -3 \\ 9 \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$

$\vec{BC} = \begin{pmatrix} 12-6 \\ -3-(-1) \\ 9-5 \end{pmatrix} = \vec{BC} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$

$K \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$

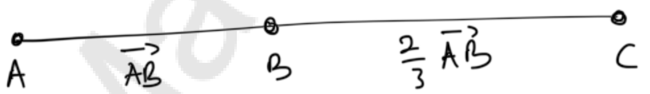
$\begin{pmatrix} 9K \\ -3K \\ 6K \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$

$9K = 6 \qquad -3K = -2 \qquad 6K = 4$

$K = \frac{6}{9} = \frac{2}{3} \qquad K = \frac{-2}{-3} = \frac{2}{3} \qquad K = \frac{4}{6} = \frac{2}{3}$

Thus AB and BC are parallel.
AB and BC Share a Common Point B.
This makes A, B and C Collinear.

(b)



$1 \quad : \quad \frac{2}{3}$

$3 : 2 = AB : BC$

6. (a) Express $5\cos x - 9\sin x$ in the form $k\cos(x+a)$ where $k>0$ and $0<a<2\pi$. **4**

(b) Hence solve $5\cos x - 9\sin x = 7$ for $0 \le x < 2\pi$. **3**

(a) $5\cos x - 9\sin x$

$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$

$k\cos(x+a) = k\cos x\cos a - k\sin x \sin a$

$5\cos x \qquad - 9\sin x$

$k\cos x\cos a = 5\cos x \qquad k\sin x\sin a = 9\sin x$

$k\cos a = 5 \qquad k\sin a = 9$

$\frac{k\sin a}{k\cos a} = \frac{9}{5} \qquad k^2\sin^2 a + k^2\cos^2 a = 9^2+5^2$

$\tan a = \frac{9}{5} \qquad k^2(\sin^2 a + \cos^2 a) = 81+25$

$k^2 \times 1 = 106$

$a = \tan^{-1}\left(\frac{9}{5}\right) \qquad k = \sqrt{106}$

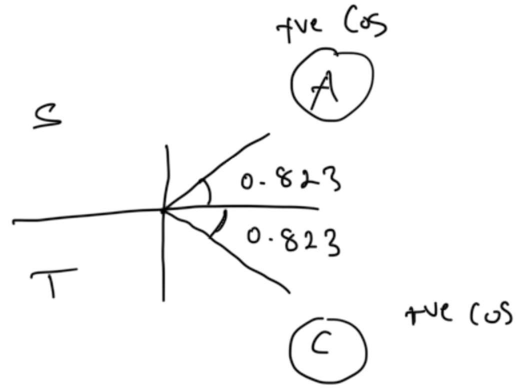
$a \simeq 1.064$ (3 d.p.) $\qquad k \approx 10.3$ (3 s.f.)

$5\cos x - 9\sin x = \sqrt{106}\cos(x+1.064)$

(b) $5\cos x - 9\sin x = 7$

$\sqrt{106}\cos(x+1.064) = 7$

$\cos(x+1.064) = \frac{7}{\sqrt{106}}$



$x + 1.064 = \cos^{-1}\left(\frac{7}{\sqrt{106}}\right)$

$x + 1.064 = 0.823$, $\quad x + 1.064 = 2\pi - 0.823$

reject $\qquad x + 1.064 = 5.46$

$x = 4.396$ (3 d.p.)

$x + 1.064 = 2\pi + 0.823$

$x = 2\pi + 0.823 - 1.064$

$x \simeq 6.042$ (3 d.p.)

MARKS

7. Find $\int (3x+2)^7 \, dx$. 2

$$\int (3x+2)^7 \, dx$$

$$u = 3x+2$$

$$\frac{du}{dx} = 3$$

$$dx = \frac{du}{3}$$

$$\int u^7 \times \frac{du}{3}$$

$$\frac{1}{3} \int u^7 \, du$$

$$\frac{1}{3} \left(\frac{u^8}{8} \right) + C$$

$$\frac{1}{3} \frac{(3x+2)^8}{8} + C$$

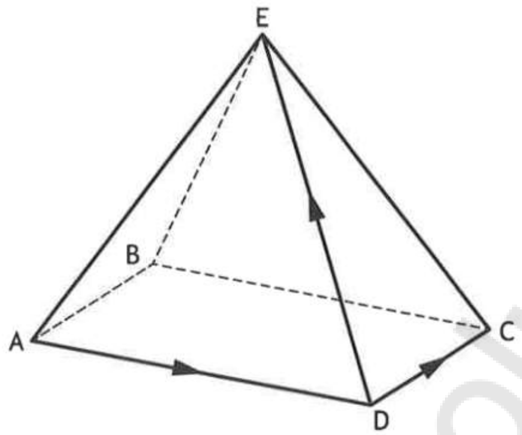
$$\frac{(3x+2)^8}{24} + C$$

8. E,ABCD is a rectangular-based pyramid as shown.

$\overrightarrow{AD} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

$\overrightarrow{DC} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$

$\overrightarrow{DE} = -4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$



Express $\overrightarrow{BE}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. 2

$\overrightarrow{AB} = \overrightarrow{DC} \qquad \overrightarrow{AD} = \overrightarrow{BC}$

$\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$

$\overrightarrow{BE} = \overrightarrow{BC} + (-\overrightarrow{DC}) + \overrightarrow{DE}$

$\overrightarrow{BE} = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ 4 \end{pmatrix}$

$\overrightarrow{BE} = \begin{pmatrix} 6 - 2 - 4 \\ 4 + 4 - 3 \\ 2 - 2 + 4 \end{pmatrix}$

$\overrightarrow{BE} = \begin{pmatrix} 0 \\ 5 \\ 4 \end{pmatrix} = \underline{\underline{5\mathbf{j} + 4\mathbf{k}}}$

9. A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 4$, where m is a constant.

(a) The sequence approaches a limit of 10 as $n \to \infty$.

Determine the value of m. **2**

(b) Given that $u_1 = 19$, calculate the value of u_0. **1**

(a)

$$m(10) + 4 = 10$$

$$10m + 4 = 10$$

$$10m = 6$$

$$m = 0.6$$

(b)

$$19 = 0.6(u_0) + 4$$

$$15 = 0.6u_0$$

$$u_0 = 25$$

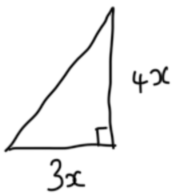
MARKS

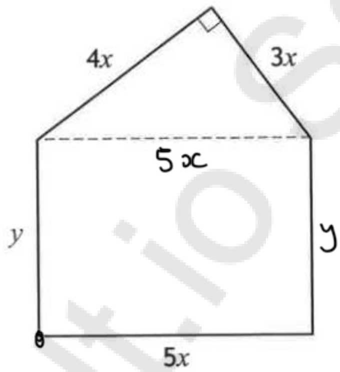
10. A hotel owner is designing signs showing the room numbers.

Each sign is a rectangle with a right-angled triangle above it.

The length and breadth of the rectangle are $5x$ centimetres and y centimetres respectively.

The shorter sides of the triangle are $3x$ centimetres and $4x$ centimetres.





The area of the sign is 150 square centimetres.

(a) Show that the perimeter, P cm, of the sign is given by

$$P = 9.6x + \frac{60}{x}.$$

3

$A_R = 5x \times y = 5xy$

$A_T = \frac{3x \times 4x}{2} = \frac{12x^2}{2} = 6x^2$

Total Area $= 5xy + 6x^2$

$150 = 5xy + 6x^2$

$150 - 6x^2 = 5xy$

$\frac{150}{5x} - \frac{6x^2}{5x} = y$

$\frac{30}{x} - \frac{6x}{5} = y$

$P = 5x + y + 3x + 4x + y$

$P = 12x + 2y$

$P = 12x + 2\left(\frac{30}{x} - \frac{6}{5}x\right)$

$P = 12x + \frac{60}{x} - \frac{12}{5}x$

$P = 9.6x + \frac{60}{x}$

Shown.

(b) Find the minimum value of P. **6**

$$P = 9.6x + \frac{60}{x}$$

$$P = 9.6x + 60x^{-1}$$

$$\frac{dP}{dx} = 9.6 - 60x^{-2}$$

$$0 = 9.6 - \frac{60}{x^2}$$

$$\frac{60}{x^2} = 9.6$$

$$x^2 = \frac{60}{9.6}$$

$$x = \sqrt{\frac{60}{9.6}}$$

$$x = \frac{5}{2}\text{ cm} \qquad x = -\frac{5}{2}\text{ cm}$$

reject

$$P_{min} = 9.6\left(\frac{5}{2}\right) + \frac{60}{5/2}$$

$$P_{min} = 48\text{ cm}$$

MARKS

11. Solve $3\sin 2x° + 4\cos x° = 0$ for $0 \leq x < 360$. 4

$$3(2\sin x\cos x) + 4\cos x = 0$$

$$6\sin x\cos x + 4\cos x = 0$$

$$6\sin x\cos^2 x + 4\cos^2 x = 0$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$6\sin x(1-\sin^2 x) + 4(1-\sin^2 x) = 0$$

$$6\sin x - 6\sin^3 x + 4 - 4\sin^2 x = 0$$

$$6\sin^3 x + 4\sin^2 x - 6\sin x - 4 = 0$$

$$3\sin^3 x + 2\sin^2 x - 3\sin x - 2 = 0$$

let $\sin x = y$

$$3y^3 + 2y^2 - 3y - 2 = 0$$

At $y = 1$ $\quad 0 = 0$

$\therefore$ $y - 1$ is a factor

$$\begin{array}{r l}
 & 3y^2 + 5y + 2 \\
y - 1 \,\big) & 3y^3 + 2y^2 - 3y - 2 \\
 & -(3y^3 - 3y^2) \\
 & 5y^2 - 3y \\
 & -(5y^2 - 5y) \\
 & 2y - 2 \\
 & -(2y - 2) \\
 & 0 \quad 0
\end{array}$$

$$(y-1)(3y^2+5y+2) = 0$$

$\frac{6}{+3,+2}$

$$(y-1)(3y^2+3y+2y+2) = 0$$

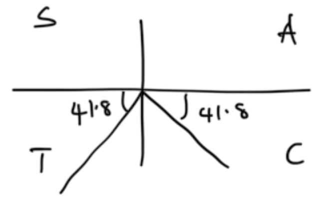
$$(y-1)(3y(y+1)+2(y+1)) = 0$$

$$(y-1)(3y+2)(y+1) = 0$$

$$y = 1 \quad y = -\frac{2}{3} \quad y = -1$$

$\sin x = 1$ $\quad \sin x = -1$ $\quad \sin x = -\frac{2}{3}$

$x = 90°$ $\quad x = 270°$



$x = 180 + 41.8$

$x = 360 - 41.8$

$x = 318.2°$

$x = 221.8°$

12. Functions f and g are defined on the set of real numbers by:

- $f(x) = x^5 + 3$
- $g(x) = 1 - x^3$.

(a) Find an expression for $h(x)$, where $h(x) = f(g(x))$. **2**

(b) Find $h'(x)$. **2**

(a) $h(x) = f(g(x))$

$h(x) = (1 - x^3)^5 + 3$

(b) $u = 1 - x^3$ $\quad \frac{du}{dx} = -3x^2$

$y = u^5 + 3$

$\frac{dy}{du} = 5u^4$

$h'(x) = \frac{dy}{dx} = 5u^4 \times -3x^2$

$h'(x) = 5(1 - x^3)^4 \times -3x^2$

$h'(x) = -15x^2(1 - x^3)^4$

13. A radioactive substance, which has been collected, decays over time.

The mass of the radioactive substance remaining is modelled by

$$M = 150e^{-0.0054t}$$

where M is the mass, in micrograms, t years after the radioactive substance was collected.

(a) Determine the initial mass of the radioactive substance. 1

(b) Calculate the time taken for the mass of the radioactive substance to decay to 120 micrograms. 4

(a) At $t=0$, $M = 150e^{-0.0054\times 0}$

$M = 150$ micrograms

(b) $120 = 150e^{-0.0054t}$

$\frac{4}{5} = e^{-0.0054t}$

$\left(\frac{5}{4}\right)^{-1} = e^{-0.0054t}$

$\frac{5}{4} = e^{0.0054t}$

$\ln\left(\frac{5}{4}\right) = \ln e^{0.0054t}$

$\ln\left(\frac{5}{4}\right) = 0.0054t \ln e$

$\ln\left(\frac{5}{4}\right) = 0.0054t$

$t = \frac{\ln\left(\frac{5}{4}\right)}{0.0054}$

$t \approx 41.3$ years (1 d.p.)

MARKS

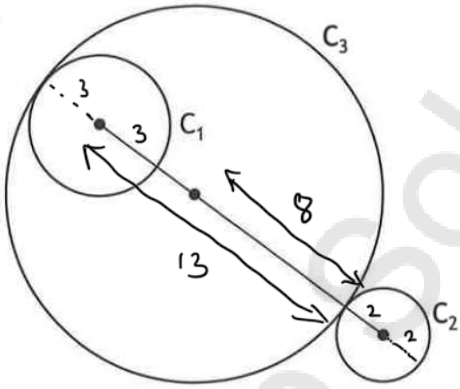
14. Circle C_1 has equation $(x+5)^2+(y-6)^2=9$.

(a) State the centre and radius of C_1. **2**

Circle C_2 has equation $x^2+y^2-14x+6y+54=0$.

(b) State the centre and radius of C_2. **2**

Circles C_1, C_2 and C_3 are touching as shown in the diagram.
The centre of circle C_3 lies on the line joining the centres of C_1 and C_2.



(c) Determine the equation of C_3. **3**

(a) Centre of circle 1 : $(-5, 6)$

$r^2=9$, $r=3$ ← radius of circle 1

(b) Centre of Circle 2

$(7, -3)$

$r^2=4$, $r=2$

radius of circle 2

$x^2+y^2-14x+6y+54=0$

$x^2-14x+y^2+6y=-54$

$(x-7)^2-(7)^2+(y+3)^2-(3)^2=-54$

$(x-7)^2+(y+3)^2=-54+49+9$

$(x-7)^2+(y+3)^2=4$

(c) $(-5,6)$ $(7,-3)$

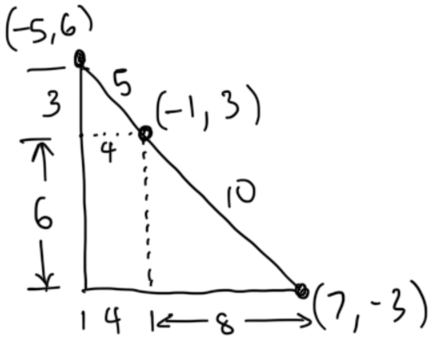
$D=\sqrt{(7-(-5))^2+(-3-6)^2}$

$D=15$

$15-2=13$

Diameter of Circle 3 $=13+3=16$

radius of circle 3 = $16 \div 2 = 8$ $C_3\ (x, y)$



Centre of circle 3 $(-1, 3)$

$$(x+1)^2 + (y-3)^2 = 64$$

[END OF QUESTION PAPER]