Surname	Centre Number	Candidate Number
Other Names		0

GCSE

3300U60-1

A17-3300U60-1

MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

MONDAY, 13 NOVEMBER 2017 – MORNING

1 hour 45 minutes

For Examiner's use only

Question	Maximum Mark	Mark Awarded
1.	3	
2.	4	
3.	6	
4.	10	
5.	5	
6.	5	
7.	6	
8.	5	
9.	7	
10.	3	
11.	4	
12.	2	
13.	3	
14.	4	
15.	3	
16.	2	
17.	8	
Total	80	

ADDITIONAL MATERIALS

A calculator will be required for this paper.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the continuation page(s) at the back of the booklet. Question numbers must be given for all work written on the continuation page.

Take π as 3·14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question **4**(*a*), the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

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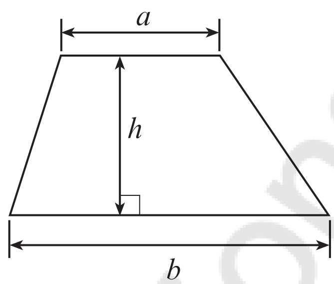
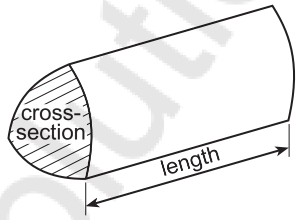
Formula List - Higher Tier

Area of trapezium = $\frac{1}{2}(a + b)h$

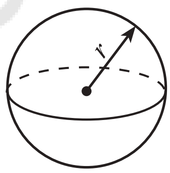


Volume of prism = area of cross-section × length



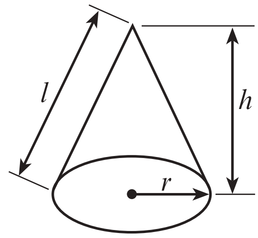
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

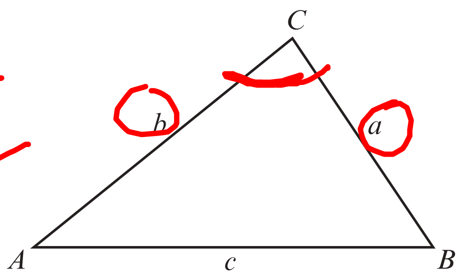


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1+\frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

$\boxed{a^m \times a^n = a^{m+n}}$

Examiner only

1. Simplify each of the following and circle the correct answer in each case.

(a) $6p^6 \times 3p^3$ $\qquad 6p^6 \times 3p^3 = 18p^{6+3} = \underline{18p^9}$ [1]

$9p^9 \qquad 9p^{18} \qquad 18p^{18} \qquad 18p^2 \qquad 18p^9$

$\boxed{\frac{a^m}{a^n} = a^{m-n}}$

(b) $3{\cdot}4g^8 \div 13{\cdot}6g^2$ [1]

$\frac{g^4}{4} \qquad \frac{g^6}{4} \qquad 4g^4 \qquad 4g^6 \qquad 0{\cdot}4g^6$

0^0

(c) $\frac{m^3 \times m^6}{m^9}$ $= \frac{m^{3+6}}{m^9} = \frac{m^9}{m^9} = m^{9-9} = m^0 = 1$ [1]

$1 \qquad m \qquad m^2 \qquad m^4 \qquad 4$

17

$$\frac{3{\cdot}4g^8}{13{\cdot}6g^2} \times \frac{10}{10} = \frac{\cancel{34}g^8}{\cancel{136}g^2} = \frac{1}{4}\frac{g^8}{g^2}$$

$$\frac{1}{4}g^{8-2} = \frac{g^6}{4} \;//$$

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 (3300U60-1) **Turn over.**

Examiner only

2. A solution of the equation

$$x^3 + 2x = 91$$

lies between 4 and 5.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

$x^3 + 2x = 91$ solution between 4, 5

Trial and improvement method

4·1, 4·2, 4·3, 4·4, 4·5, 4·6, 4·7, 4·8, 4·9

when $x = 4{\cdot}1$ 91 ✗

$x^3 + 2x = 4{\cdot}1^3 + 2 \times 4{\cdot}1 = 68{\cdot}921 + 8{\cdot}2 = 77{\cdot}121$

when $x = 4{\cdot}2$

$x^3 + 2x = 4{\cdot}2^3 + 2 \times 4{\cdot}2 = 74{\cdot}088 + 8{\cdot}4 = 82{\cdot}488$

when $x = 4{\cdot}3$

$x^3 + 2x = 4{\cdot}3^3 + 2 \times 4{\cdot}3 = 79{\cdot}507 + 8{\cdot}6 = 88{\cdot}107$

when $x = 4{\cdot}4$

$x^3 + 2x = 4{\cdot}4^3 + 2 \times 4{\cdot}4 = 85{\cdot}184 + 8{\cdot}8 = 93{\cdot}984$

when $x = 4{\cdot}5$

$x^3 + 2x = 4{\cdot}5^3 + 2 \times 4{\cdot}5 = 91{\cdot}125 + 9 = 100{\cdot}125$

4·349495

So, $x = 4{\cdot}3$ and $x = 4{\cdot}4$ is the closest to 91

4·35 4·35 4·35

when x

$x^3 + 2x = 4{\cdot}35^3 + 2 \times 4{\cdot}35 = 82{\cdot}313 + 8{\cdot}7 = 91{\cdot}013$

$x = 4{\cdot}35$

$x = \boxed{4{\cdot}3}$

 (3300U60-1)

Examiner only

3. *ABC* is an isosceles triangle with *AB* = *AC*.

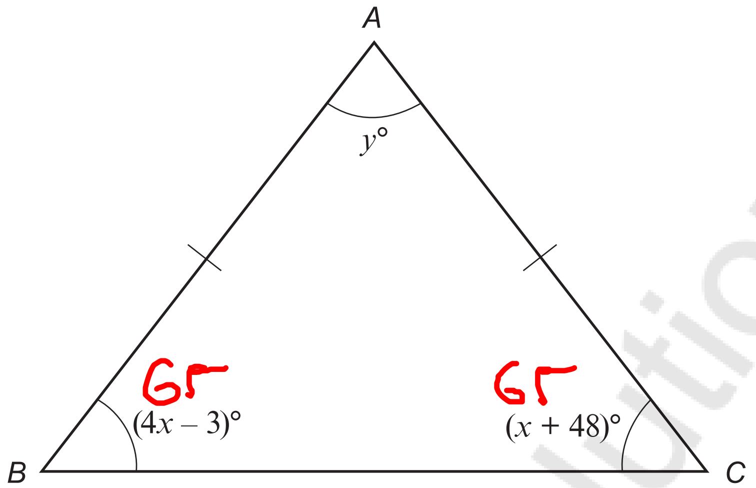

Diagram not drawn to scale

Calculate the value of y. [6]

Since ΔABC is isosceles

then $\angle B = \angle C$

$4x - 3 = x + 48$

$-x + 3 \quad -x + 3$

$3x = 51$

$x = 51/3 = 17$

$\angle B = 4x - 3 = 4 \times 17 - 3 = 65$

$\angle C = x + 48 = 17 + 48 = 65$

$y + 65 + 65 = 180$ [Sum of angle in a triangle]

$y + 130 = 180$

$-130 \quad -130$

$y = 50°$

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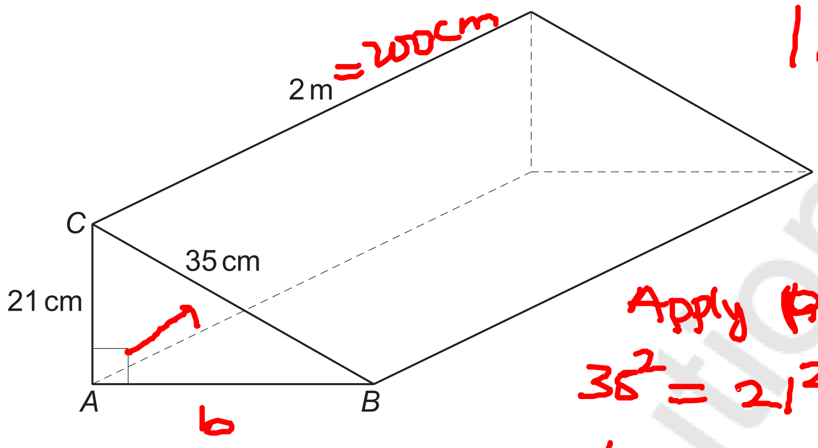
 (3300U60-1) **Turn over.**

Examiner only

4. A triangular prism of length 2 metres is shown below.



1m = 100cm

2m = 200cm

Apply Pythagoras theorem

$35^2 = 21^2 + b^2$

$1225 = 441 + b^2$

$-441 \quad -441$

$784 = b^2$

$b = \sqrt{784}$

$b = 28\text{cm}$

Diagram not drawn to scale

$AC = 21\,\text{cm}$, $BC = 35\,\text{cm}$ and $B\hat{A}C = 90°$.

(a) *In this part of the question, you will be assessed on the quality of your organisation, communication and accuracy in writing.*

Calculate the area of triangle *ABC*.
Give your answer in cm^2.
You must show all your working. [5 + 2 OCW]

Area of triangle $= \frac{1}{2} b \times h$

$= \frac{1}{2} \times 28 \times 21$

Area $= 294\,\text{cm}^2$

Examiner only

(b) Calculate the volume of the prism.
You must give the units of your answer. [3]

Volume = Base Area × height

Volume = 294 × 200

= 58,800 cm^3

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 (3300U60-1)

Turn over.

Examiner only

5. Find the answer to the following number problem. [5]

'(the LCM of 12, 18 and 24) ÷ (the HCF of 36 and 54)'.

LCM of 12 18 24

2	12	18	24
2	6	9	12
2	3	9	6
3	3	9	3
3	1	3	1
	1	1	1

3
9

$LCM = 2 \times 2 \times 2 \times 3 \times 3 = 72$

HCF of 36 and 54

18
9

$36 = 2 \times 2 \times 3 \times 3$

3

$54 = 2 \times 3 \times 3 \times 3$

27

$HCF = 2 \times 3 \times 3 = 18$

$\frac{LCM}{HCF} = \frac{72}{18} = 4$

4

 (3300U60-1)

Examiner only

6. *(a)* Rearrange the following formula to make x the subject.
Give your answer in its simplest form. [3]

$$2(x + y) = 7y - 3$$

$2(x+y) = 7y - 3$

$2x + 2y = 7y - 3$

$-2y \quad -2y$

$2x = 5y - 3$

$\frac{2x}{2} = \frac{5y-3}{2}$

$x = \frac{5y-3}{2}$

(b) Write down the nth term of the following sequence. [2]

3, $+3$ 6, $+5$ 11, $+7$ 18, $+9$ 27, …

AP or GP

3, 6, 11, 18, 27

-2 1 4 9 16 25

$1^2 \quad 2^2 \quad 3^2 \quad 4^2 \quad 5^2$

nth term $= n^2 + 2$

$5^2 + 2 = 25$

$1^2 + 2 = 3$ $\quad 3^2 + 2 = 11$

$2^2 + 2 = 6$ $\quad 4^2 + 2 = 18$

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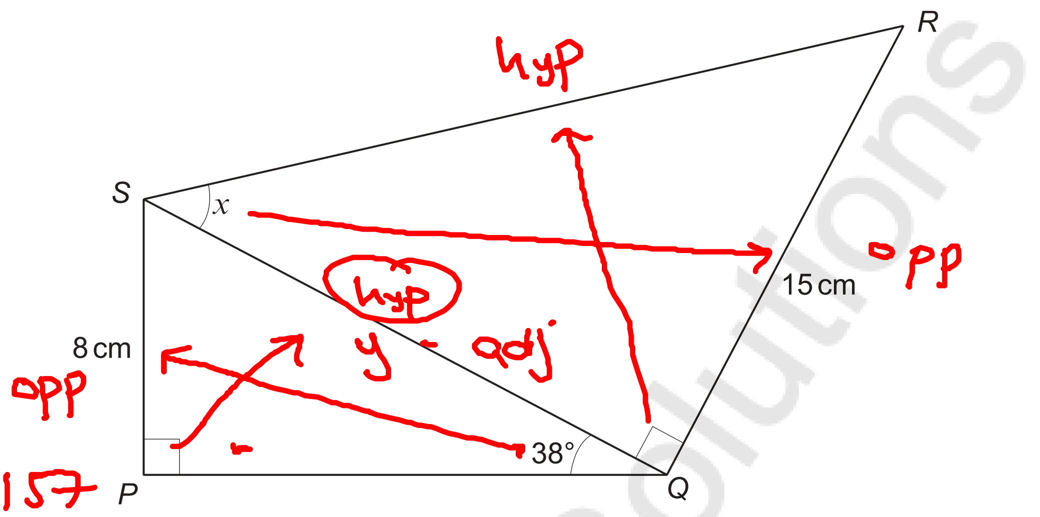
 (3300U60-1) Turn over.

Examiner only

7. The diagram shows two right-angled triangles, joined together along a common side.

$S\hat{P}Q = 90°$, $S\hat{Q}R = 90°$, $S\hat{Q}P = 38°$, $PS = 8$ cm and $QR = 15$ cm.



$\sin 38 = 0.6157$

Diagram not drawn to scale

Calculate the size of angle x. [6]

Apply Trig [SOH] CAH $\boxed{TOA}$

$\sin\theta = \dfrac{opp}{hyp}$

$\sin 38 = \dfrac{8}{y}$

$\sin 38 \times y = 8$

$y = \dfrac{8}{\sin 38} = \dfrac{8}{0.6157} = 12.99$ cm

$\tan\theta = \dfrac{opp}{adj}$

$\tan x = \dfrac{15}{12.99} = 1.1547$

$\tan x = 1.1547$

$x = \tan^{-1}(1.1547) = 49.11°$

$x = 49°$

 (3300U60-1)

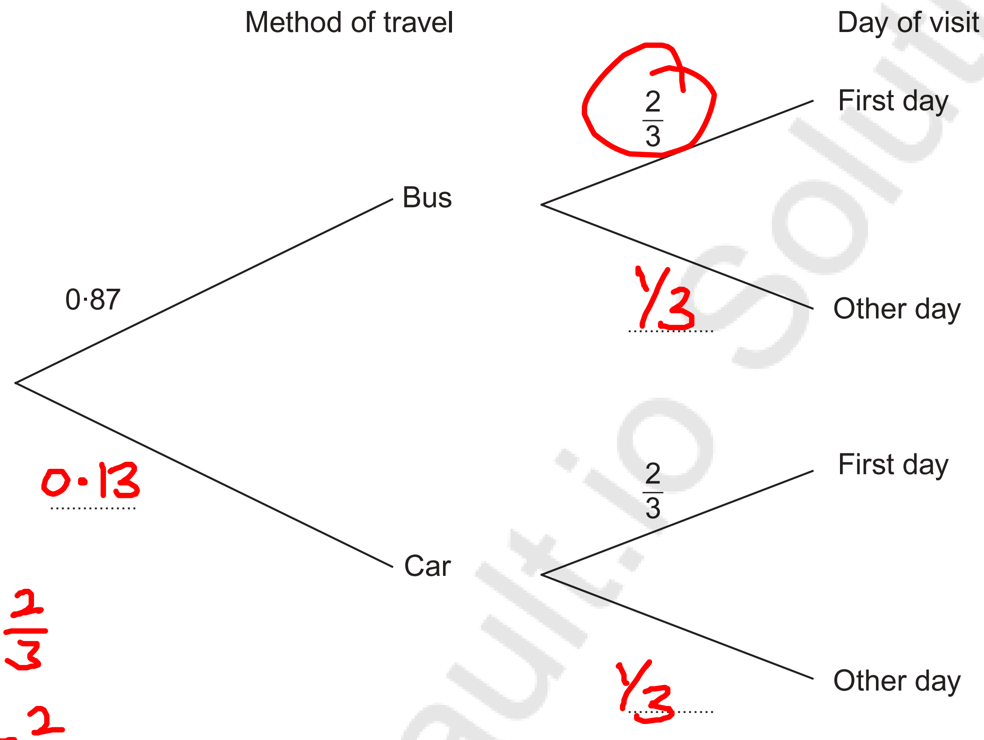
Examiner only

8. All the members of a farming club visited the Royal Welsh Agricultural Show. They all travelled to the show either by bus or by car. None of them visited the show on more than one day. The decision to travel by car or by bus was independent of the day of the visit.

A member of the club was selected at random. The probability that this member travelled by bus was 0·87.

The probability that this member visited the show on the first day was $\frac{2}{3}$.

(a) Complete the tree diagram shown below. [2]



$1 - \frac{2}{3}$

$\frac{3}{3} - \frac{2}{3}$

$\frac{3-2}{3}$

$\frac{1}{3}$

(b) What is the probability that a member, chosen at random, was **not** one of those who travelled by bus on the first day of the show? [3]

Pr (Travel by bus on first day) = Pr(Bus) × Pr(First day)

$= 0{\cdot}87 \times \frac{2}{3}$

$= 0{\cdot}58$

Pr (not Travel by bus on First day) $= 1 - 0{\cdot}58$

$= 0{\cdot}42$

 (3300U60-1) **Turn over.**

Examiner only

9. *(a)* Show that $(10w + 3)(w - 1) - (2 - 3w)^2 \equiv w^2 + 5w - 7$. [4]

$(10w+3)(w-1) - (2-3w)^2$

$10w(w-1) + 3(w-1) - [(2-3w)(2-3w)]$

$10w^2 - 10w + 3w - 3 - [2(2-3w) - 3w(2-3w)]$

$10w^2 - 7w - 3 - [4 - 6w - 6w + 9w^2]$

$10w^2 - 7w - 3 - 4 + 6w + 6w - 9w^2$

$-w$ $\quad 10w^2 - 9w^2 - 7w + 6w + 6w - 3 - 4$

$w^2 + 5w - 7$ //

LHS = RHS proved

(b) Use the quadratic formula to solve the equation $w^2 + 5w - 7 = 0$.
Give your answers correct to 2 decimal places. [3]

$1w^2 + 5w - 7 = 0 \quad ax^2 + bx + c = 0$

$a = 1 \quad b = 5 \quad c = -7$

$w = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \dfrac{-5 \pm \sqrt{5^2 - 4 \times 1 \times -7}}{2 \times 1}$

$w = \dfrac{-5 \pm \sqrt{25 + 28}}{2} = \dfrac{-5 \pm \sqrt{53}}{2}$

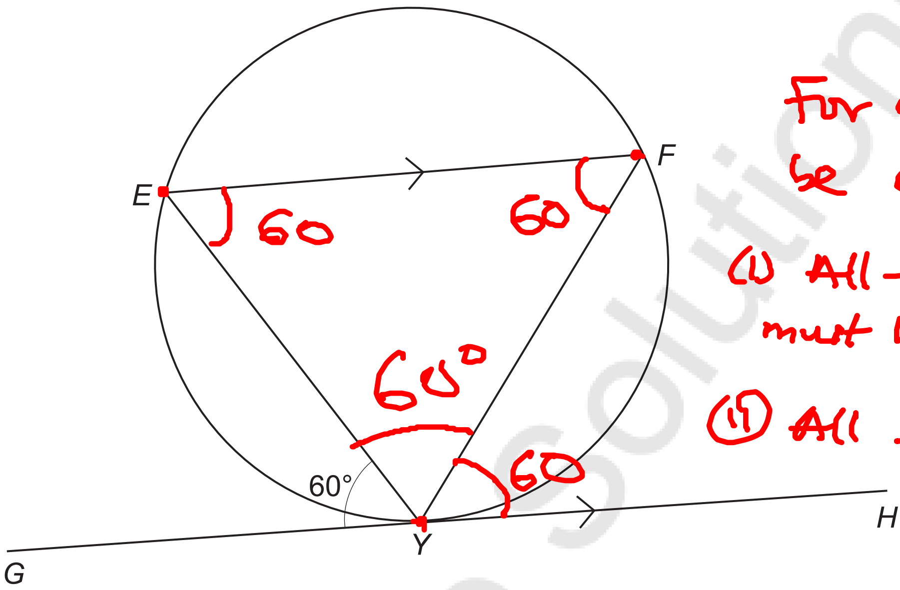
$w = \dfrac{-5 \pm 7.28}{2}$

$w = \dfrac{-5 + 7.28}{2}$ OR $\dfrac{-5 - 7.28}{2}$

$w = 1.14$ OR -6.14

Examiner only

10. The line GH is a tangent to the circle at point Y.
The line EF is parallel to the line GH.
The vertices of triangle EFY lie on the circle.
$E\hat{Y}G = 60°$.



For a triangle to be equilateral
(i) All three sides must be equal
or
(ii) All three angles must 60°

Diagram not drawn to scale

Prove that EFY is an equilateral triangle.
Give a reason for each step to justify your proof. [3]

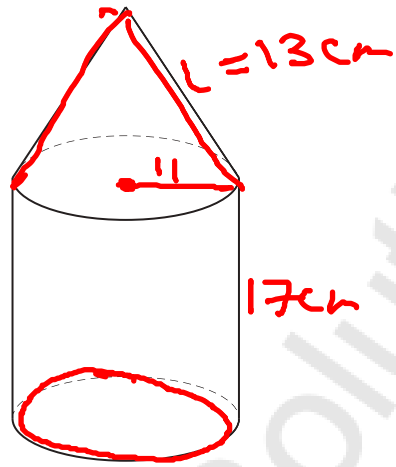
$\angle EFY = 60°$ [Alternative Segment theorem]
$\angle FYH = \angle EFY$ [Alternative angle Z]
$\angle FYH = 60°$
$\angle FEY = \angle FYH$ [Alternative Segment theorem
$\angle FEY = 60$
$\angle E + \angle F + \angle Y = 180$ [Sum of angle in a triangle]
$60 + 60 + \angle Y = 180$
$120 + \angle Y = 180$
$-120 \qquad -120$
$\angle Y = 60$

So, since all the angles in the triangle are equal (60°). Then, triangle EFY is an equilateral triangle.

Turn over.

Examiner only

11. A cone is joined to a cylinder, as shown below.
The cone has a base radius of 11 cm and a slant height of 13 cm.
The cylinder has the same radius, 11 cm, and a height of 17 cm.
Calculate the **total** surface area of the composite solid. [4]



r = 11 cm
L = 13 cm
h = 17 cm

Diagram not drawn to scale

Total Surface = Area of Circle + Curved surface area of cylinder + Curved surface area of Cone

Total Surface Area = $\pi r^2 + 2\pi rh + \pi rl$

TSA = $3.14 \times 11^2 + 2 \times 3.14 \times 11 \times 17 + 3.14 \times 11 \times 13$

TSA = $379.94 + 1174.36 + 449.02$

TSA = 2003.32 cm^2

TSA = 2003 cm^2

Total surface area = ... cm^2

 (3300U60-1)

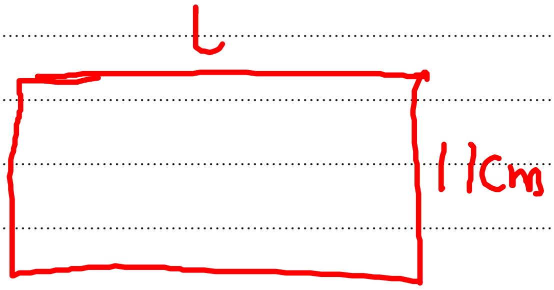
Examiner only

12. The area of a rectangle is 137 cm^2, correct to the nearest cm^2.
Its width is 11 cm, correct to the nearest cm.

Calculate the greatest possible length of the rectangle.
Give your answer correct to 3 significant figures. [2]

Range of Area
136.5 → 137.5

Range of width
10.5 cm – 11.5 cm



Area = 137 cm^2

Area = L × W

$L = \frac{\text{Area}}{W} = \frac{A}{W}$

A → greatest A
W → least width

greatest Area = 137.5 cm
Least width = 10.5 cm

L = 137.5 ÷ 10.5 = 13.1 cm to 3 s.f.

13. A bag contains 5 red counters and 5 blue counters.
Three counters are drawn at random from the bag at the same time.
Calculate the probability that the three counters will be the same colour. [3]

Red — 5
Blue — 5

Pr(three counter be red) + Pr(three counter be blue)

$R_1 R_2 R_3 + B_1 B_2 B_3$

$Pr(R_1) = \frac{5}{10}$ $Pr(R_2) = \frac{4}{9}$ $Pr(R_3) = \frac{3}{8}$

$Pr(B_1) = \frac{5}{10}$ $Pr(B_2) = \frac{4}{9}$ $Pr(B_3) = 3/8$

$R_1 R_2 R_3 + B_1 B_2 B_3$

$\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} + \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{6}$

0.083 + 0.083

0.167

  Turn over.

Examiner only

14. *(a)* Sketch the curve $y = \sin x$, for values of x in the range $x = 0°$ to $x = 360°$. [1]

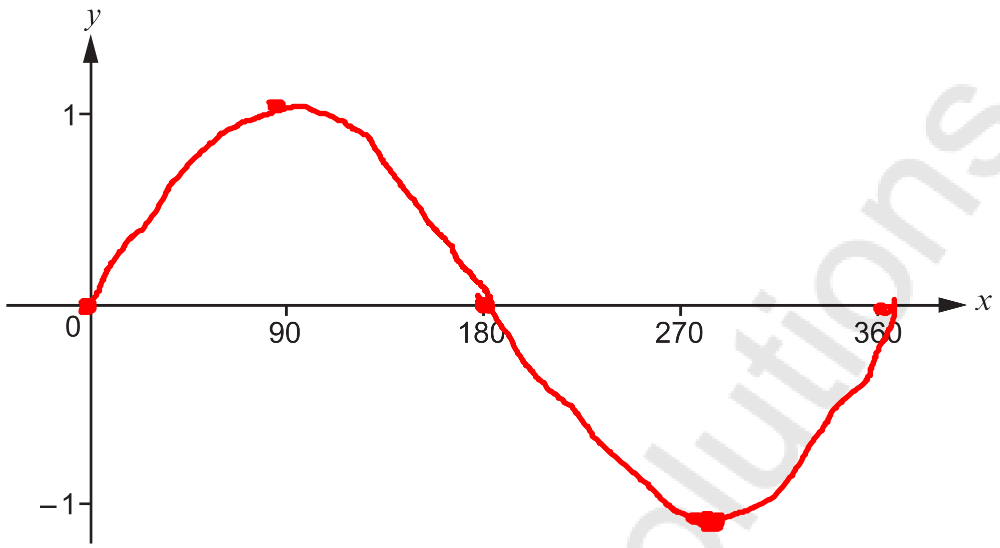
$\sin 0 = 0$

$\sin 90 = 1$

$\sin 180 = 0$

$\sin 270 = -1$

$\sin 360 = 0$



(b) Solve each of the following equations.
Give all answers in the range $x = 0°$ to $x = 360°$.

0, to 360°

(i) $\sin x = 0{\cdot}3$ [2]

$\sin x = 0{\cdot}3$

$x = \sin^{-1}(0{\cdot}3)$

$x = 17{\cdot}46°$ ✓

$\theta_n = 180n + (-1)^n\theta$

$x_n = 180n + (-1)^n x$

$x_1 = 180 + (-1)^1 17{\cdot}46$

$x_1 = 180 - 17{\cdot}46 = 162{\cdot}54°$ ✓

$x_2 = 180 \times 2 + (-1)^2 17{\cdot}46$

$= 360 + 17{\cdot}46 = 377{\cdot}46$ ✱

Ans

$x = 17{\cdot}46$

$x = 162{\cdot}54$

$n = 1$

(ii) $\sin x + 1 = 0$ [1]

0, 360

$\sin^{-1}(-1) = -90°$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \sin^{-1}(-1) = 270°$ ✓

$x_n = 180n + (-1)^n x$

$x_1 = 180 + (-1)^1(-90) = 180 + 90 = 270$

$x_2 = 360 + (-1)^2(-90) = 360 - 90 = 270$

$x = 270$

Examiner only

15. Two **similar** pyramids have volumes of 3970 cm^3 and 3100 cm^3 respectively.
The height of the larger pyramid is 25 cm.
Calculate the height of the smaller pyramid. [3]

Pyramid (1) Volume = 3970 cm^3

Pyramid (2) Volume = 3100 cm^3

Volume of a Pyramid = $\frac{1}{3}$ Base Area × height

Scale factor $K = \sqrt[3]{\frac{3970}{3100}} = \sqrt[3]{1.281}$

$K = 1.086$

height of smaller pyramid = $\frac{h}{K} = \frac{25}{1.086}$

Height = ~~23.02~~ cm

$h_{smaller} = \underline{23 cm}$

 (3300U60-1) **Turn over.**

Examiner only

16. Each of the two graphs below is described by **one** of the equations on the right.
Put a **tick** in the box next to the equation which correctly describes each graph. [2]

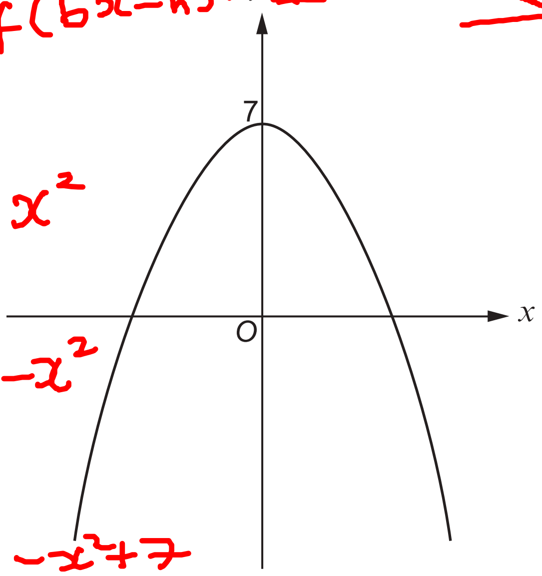
Graph A

$y = af(bx-h)+k$ $\quad$ $y = x^2$

x^2

$-x^2$

$-x^2+7$

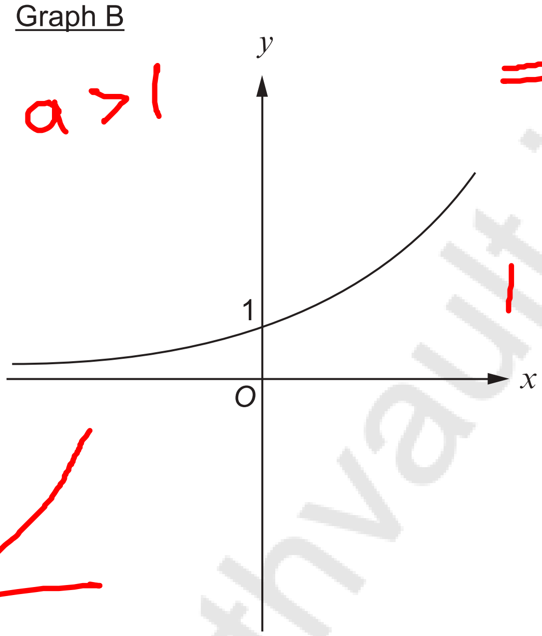


	Equation describing graph A
$y = 7x^2$	
$y = -(x+7)^2$	
$y = (x-7)^2$	
$\boxed{y = 7 - x^2}$	✓
$y = x^2 + 7$	

$y = a^x$ $\quad$ $a \neq 0$ $\quad$ $a > 1$

Graph B

a^x $\quad$ $a > 1$



$y=2^x$

1

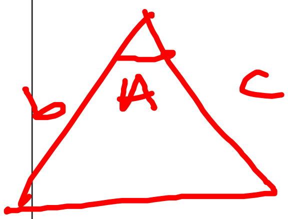
	Equation describing graph B
$y = x^2 + 1$ ✗	
$y = 2^x$	✓
$y + 1 = x^2$ ✗	
$y = \frac{1}{x}$ ✗	
$y = x^0$	

 (3300U60-1)

Examiner only

17. *ABC* represents the **sector** of a circle with radius 7 cm and centre *A*, as shown below.
$B\hat{A}C = x°$, *AD* = 3 cm and *BD* = 6 cm.

Area of triangle $= \frac{1}{2} bc \cos x$



$\frac{1}{2} bc \cos A$

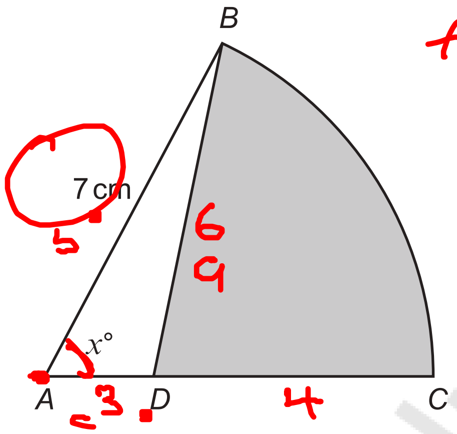


Diagram not drawn to scale

Find the area of the shaded region *BCD*. [8]

$a^2 = b^2 + c^2 - 2bc\cos\theta$

$6^2 = 7^2 + 3^2 - 2 \times 7 \times 3 \cos x$

$36 = 49 + 9 - 42\cos x$

$36 = 58 - 42\cos x$

$36 - 58 = -42\cos x$

$-22 = -42\cos x$

$\frac{-22}{-42} = \cos x$

$\cos x = 0.5238$

$x = \cos^{-1}(0.5238)$

$x = 58.41°$

Area of Sector $ABC = \frac{\theta}{360} \times \pi r^2$

$= \frac{58.41}{360} \times 3.14 \times 7^2$

$= 24.96\ \text{cm}^2$

END OF PAPER

$D \cdot 8518$

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.
	Area of triangle BAD $= \frac{1}{2} AB \times AD \times \sin A$
	Area of triangle BAD $= \frac{1}{2} \times 7 \times 3 \times \sin 58 \cdot 41$
	Area of triangle BAD $= \frac{1}{2} \times 7 \times 3 \times 0 \cdot 8518$
	$= 8 \cdot 94 \text{ cm}^2$
	Shaded part = Area of Sector ABC − Area of ΔBAD
	$= 24 \cdot 96 - 8 \cdot 94$
	$= \underline{16 \cdot 02 \text{ cm}^2}$
	Shaded Area $\equiv \underline{16 \text{ cm}^2}$

Examiner only

 (3300U60-1)