Surname	Centre Number	Candidate Number
First name(s)		0

GCSE

3300U50-1

A20-3300U50-1

MONDAY, 9 NOVEMBER 2020 – MORNING

MATHEMATICS
UNIT 1: NON-CALCULATOR
HIGHER TIER

1 hour 45 minutes

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space use the additional page at the back of the booklet. Question numbers must be given for all work written on the additional page.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question **4**, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1.	**5**	
2.	**3**	
3.	**6**	
4.	**6**	
5.	**3**	
6.	**5**	
7.	**6**	
8.	**4**	
9.	**3**	
10.	**6**	
11.	**2**	
12.	**3**	
13.	**3**	
14.	**5**	
15.	**4**	
16.	**4**	
17.	**2**	
18.	**2**	
19.	**8**	
Total	**80**	

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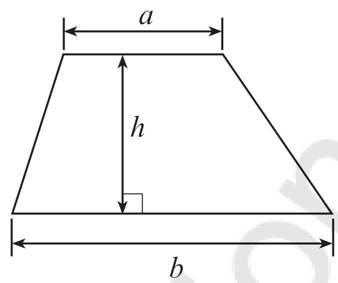
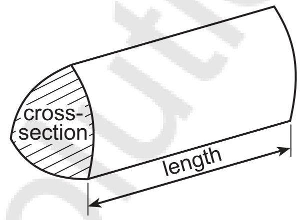
Formula List - Higher Tier

Area of trapezium = $\frac{1}{2}(a+b)h$

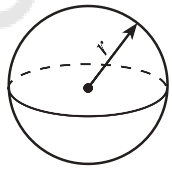


Volume of prism = area of cross-section × length



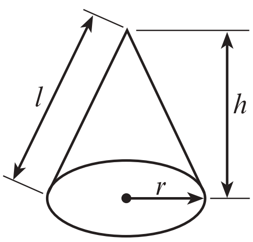
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

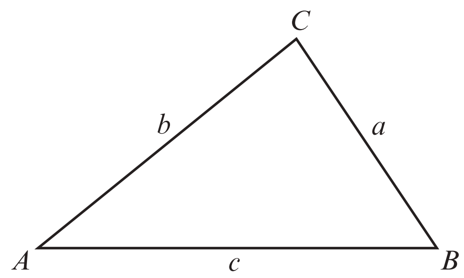


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1+\frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

Examiner only

1. *(a)* Write an expression for the nth term of the following sequence. [2]

$2 \xrightarrow{5} 7 \xrightarrow{5} 12 \xrightarrow{5} 17$ ....

$a \Rightarrow$ First term $= 2$; Common difference $= 5 \Rightarrow d$

$T_n = a + (n-1)d = 2 + (n-1)5 = 2 + 5n - 5 = 5n - 5 + 2$

$= \boxed{5n - 3}$

nth term = $5n - 3$

(b) The first four diagrams in a sequence are shown below.

$1, 3, 6, 10, 15, 21, 28$ ....

$1, 2, 3, 4, 5, 6, 7$

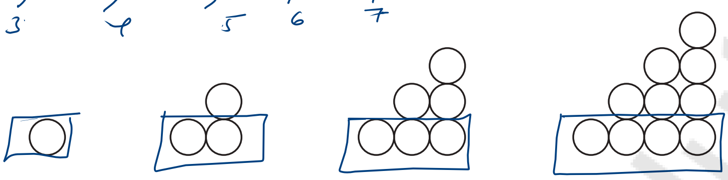


Diagram 1 Diagram 2 Diagram 3 Diagram 4 ... Diagram 17

Complete the following subtraction. [1]

$(17 + 16 + 15 + 14 + 13 \ldots + 1) - (16 + 15 + 14 + 13 \ldots + 1)$

Number of circles in Diagram 17	–	Number of circles in Diagram 16	=	17

(c) The first three diagrams in another sequence are shown below.

4 6 8

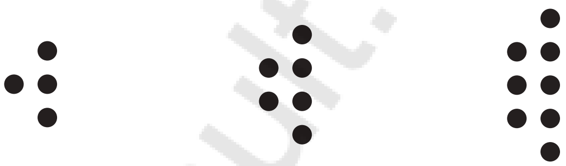


Diagram 1 Diagram 2 Diagram 3

Give an expression, in terms of n, for the number of dots (●) in Diagram n.
You must simplify your expression. [2]

$\Rightarrow 4, 6, 8$; $a = 4, d = 2$

$T_n = a + (n-1)d = 4 + (n-1)2 = 4 + 2n - 2$

$T_n = 2n + 4 - 2 = 2n + 2$ or $2(n+1)$

$T_n = 2n + 2$ or $2(n+1)$

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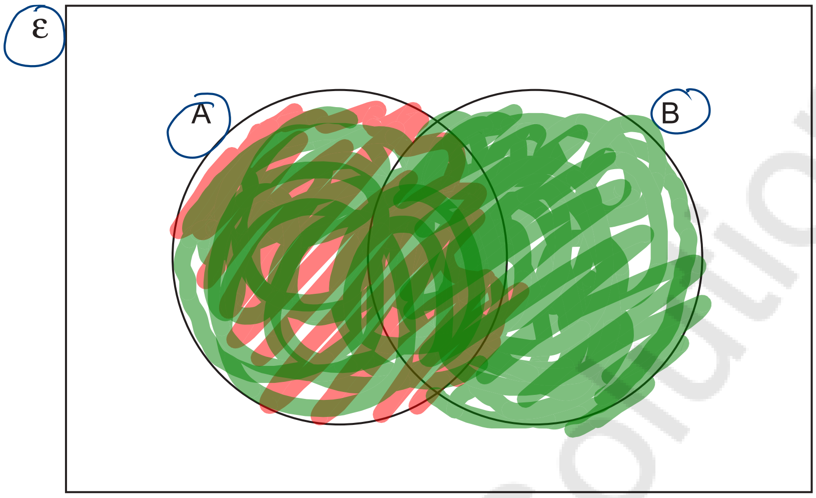
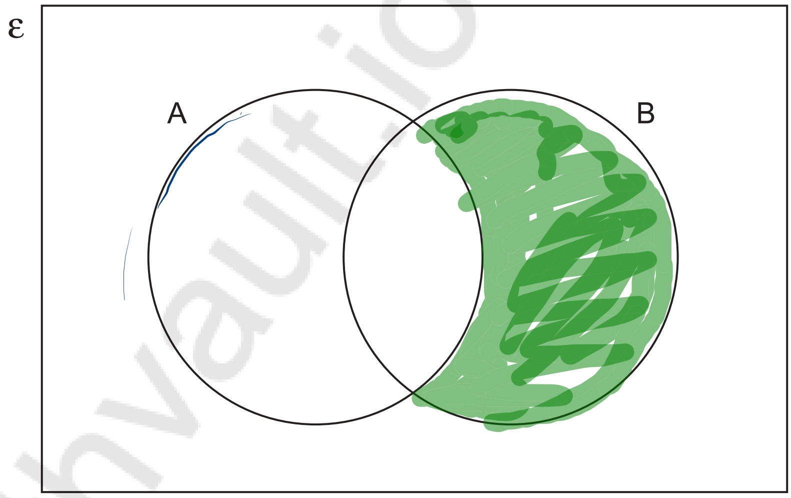
2. *(a)* On each Venn diagram, shade the region that represents the given set.

(i) $A \cup B$ [1]



(ii) $\cancel{A'} \cap B$ $A^c \cap B$ [1]



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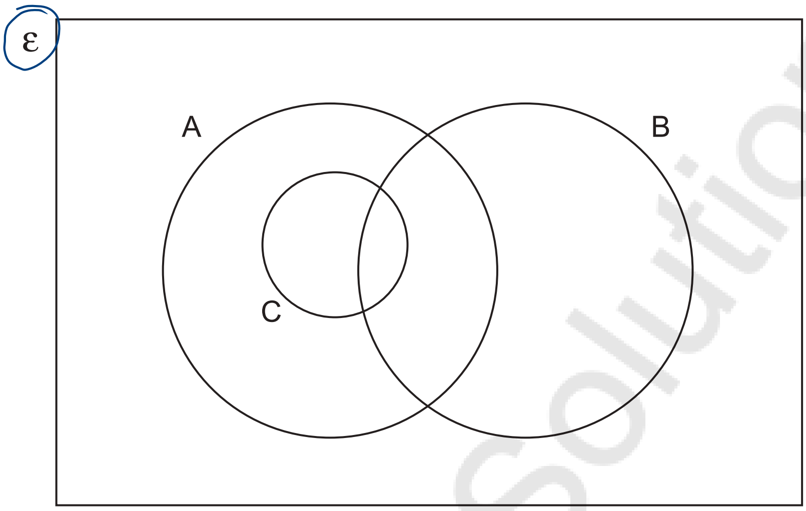
(b) In the Venn diagram below:

- Set A = multiples of 3, = 3, 6, 9, 12, 15, 18, 21, ---
- Set B = multiples of 5, = 5, 10, 15, 20, 25, 30, ---
- Set C = multiples of 6. = 6, 12, 18, 24, 30, 36, ---



Explain why the circle representing Set C is drawn inside the circle drawn to represent Set A. [1]

⇒ The reason is because 3 is a factor of 6.

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Turn over.

Examiner only

3. The table below shows some of the values of $y = x^2 - 4x - 3$ for values of x from –2 to 5.

x	–2	–1	0	1	2	3	4	5
$y = x^2 - 4x - 3$	9	2	–3	–6	–7	–6	–3	2

(a) Complete the table by finding the value of y for x = −2 and the value of y for x = 2. [2]

Given $y = x^2 - 4x - 3$

When $x = -2$ | When $x = 2$

$y = (-2)^2 - 4(-2) - 3$ | $y = (2)^2 - 4(2) - 3$

$y = 4 + 8 - 3$ | $= 4 - 8 - 3$

$y = 12 - 3$ | $= -4 - 3$

$y = 9$ | $= -7$

$(x = -2, \; y = 9)$ When $(x = 2, \; y = -7)$

(b) On the graph paper opposite, draw the graph of $y = x^2 - 4x - 3$ for values of x from –2 to 5. [2]

(c) Draw the line $y = 1$ on the graph paper.
Write down the values of x where the line $y = 1$ cuts the curve $y = x^2 - 4x - 3$. [2]

Values of x are -0.8 and 4.8

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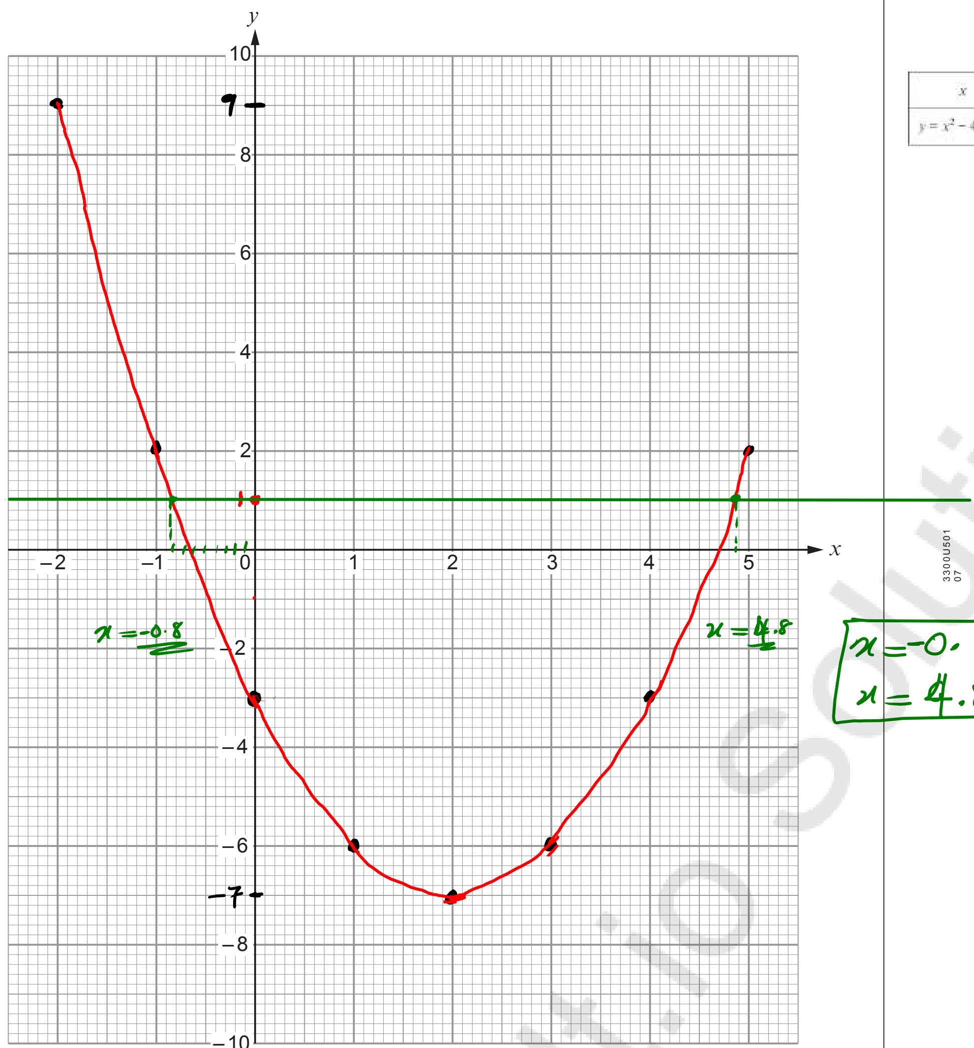

x	−2	−1	0	1	2	3	4	5
$y = x^2 - 4x - 3$	9	2	−3	−6	−7	−6	−3	2

$y = 1$

$\boxed{\begin{array}{l} x = -0.8 \\ x = 4.8 \end{array}}$ $\begin{array}{l} \to -0.85 \\ \to 4.85 \end{array}$

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 (3300U50-1) **Turn over.**

Examiner only

4. *In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.*

A sum of money is shared in the ratio 3 : 4 : 7.
The smallest share is £210.

What is the total amount of money shared?
You must show all your working. [4 + 2 OCW]

$\Rightarrow$ £210 ÷ 3 $\Rightarrow$ £70

Total amount $\Rightarrow$ $(3+4+7) \times$ £70
$\Rightarrow$ $(7+7) \times$ £70
$\Rightarrow$ $14 \times$ £70
$\Rightarrow$ £980

Examiner only

5. Find four **different** positive whole numbers so that:

- their mean is 8,
- their range is 8,
- their median is 8.

Write your four numbers in the boxes below. [3]

$\boxed{9\ \ 7} \Rightarrow 16$

$\boxed{5\ \ 11} \Rightarrow 16$

$\boxed{3\ \ 13} \Rightarrow 16$

$\boxed{6\ \ 10}$

$\boxed{4, 7, 9, 12}$

a, b, c, d

Range: $d - a = 8$; $\frac{b+c}{2} = 8$; med

$\frac{a+b+c+d}{4} = 8$ $\quad \rightarrow \boxed{b+c} = 8 \times 2 = 16$

Recall $d - a = 8$

$a + 16 + d = 8 \times 4 = 32$ $\quad d - 4 = 8$

$a + d + 16 = 32$ $\quad d = 8 + 4 = 12$

$a + d = 32 - 16 = 16$

$a + d = 16$

$a + 8 + a = 16$

$2a + 8 = 16$

$\frac{2a}{2} = \frac{8}{2}$; $a = 4$

The four numbers are $\boxed{4}$ $\boxed{7}$ $\boxed{9}$ $\boxed{12}$

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Turn over.

Examiner only

6. *(a)* Factorise $x^2 - 7x + 12$, and hence solve $x^2 - 7x + 12 = 0$. [3]

$x^2 - 7x + 12$

$x^2 - 4x - 3x + 12$

$x(x-4) - 3(x-4)$

$(x-3)(x-4)$

$(x-3)(x-4) = 0$

$x - 3 = 0$ or $x - 4 = 0$

$x = 0 + 3$ or $x = 0 + 4$

$x = 3$ or 4

12

4×3

$\boxed{-4 \quad \cancel{\times} -3}$

(b) Expand and simplify $(5x - 2)^2$. [2]

$(5x-2)^2 = (5x-2)(5x-2)$

$= 5x(5x-2) - 2(5x-2)$

$= 25x^2 - 10x - 10x + 4$

$= 25x^2 - 20x + 4$

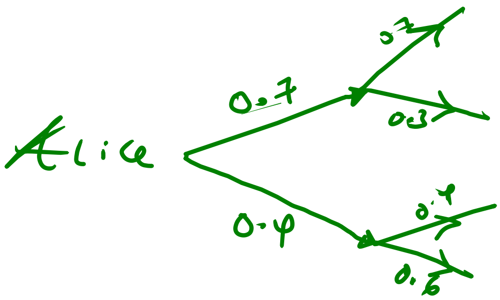
Examiner only

7. Alice works for an engineering company.

A working day is chosen at random.
From keeping a record over the last year, Alice knows that, for this working day,

- the probability that she travels to work by car is 0·7,
- the probability that she arrives at work before 8:00 a.m. is 0·4,
- her time of arrival is independent of how she travels to work.

(a) Using the above information, draw and fully label a complete tree diagram.
You must include all probabilities. [4]



(b) What is the probability that, on the randomly-chosen working day, Alice travels to work by car and arrives before 8:00 a.m.? [2]

$0.7 \times 0.4 = 0.28$

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8. A circle, centre O, has a radius of 4 cm.
A and B are points on the circumference of the circle.
Lines PA and PB are both tangents to the circle.
PB = 12 cm.

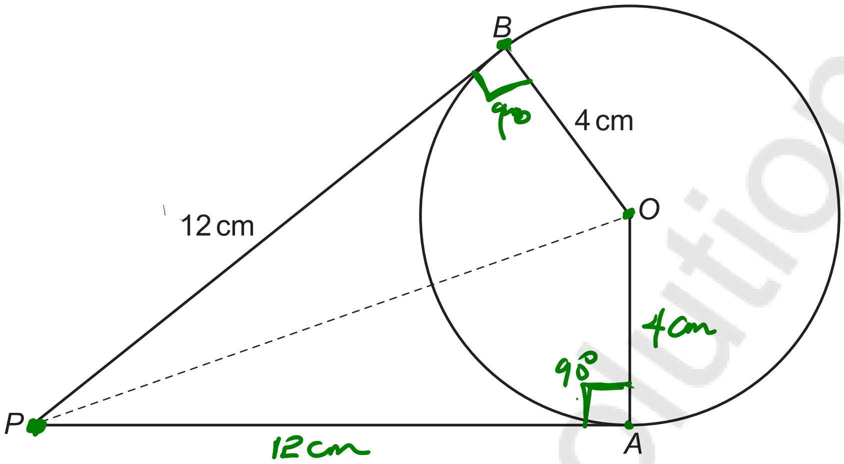


Diagram not drawn to scale

(a) What is the length of PA?
State the circle theorem you have used to find your answer. [1]

PA = 12cm

Circle theorem: tangent from an external point P are equal in length ($\overline{PB} = \overline{PA}$)

(b) What is the size of $P\hat{A}O$?
State the circle theorem you have used to find your answer. [1]

$P\hat{A}O$ = 90°

Circle theorem: the tangent at any point on a circle is perpendicular to the radius at that point

(c) Calculate the area of the quadrilateral $PAOB$. [2]

Since the quadrilateral PAOB comprises of 2 triangle $\Rightarrow$ $\text{Area}_P = 2 \times [\text{Area}_T]$

$\text{Area}_P = 2 \times \left[\frac{1}{2} \times b \times h\right] = 2 \times \left[\cancel{\tfrac{1}{2}} \times 12 \times \cancel{4}^{\,2}\right]$

$= 2 \times 24 = \underline{\underline{48\text{ cm}^2}}$

Examiner only

9. *(a)* Which one of the following equations represents a straight line that is parallel to the line $2y = 5x - 4$?
$y = mx + c$
Circle your answer. [1]

$y = 2{\cdot}5x + 3$ $\qquad$ $y = 5x - 2$ $\qquad$ $y = 0{\cdot}4x - 4$ $\qquad$ $y = -0{\cdot}4x - 2$ $\qquad$ $2y = -5x + 4$

from $\frac{2y}{2} = \frac{5x}{2} - \frac{4}{2}$; $y = \frac{5}{2}x - 2 \Rightarrow y = 2.5x + 3$

Slope $= 2.5$

(b) Which one of the following equations represents a straight line that intersects the line $y = 7x - 5$ on the y-axis?
Circle your answer. [1]

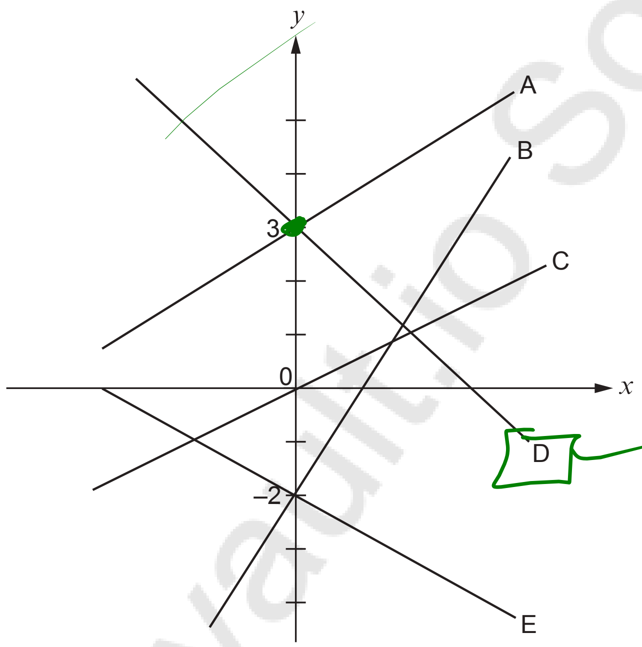
$y = mx + c$

$y = 7x + 5$ $\qquad$ $y = 5 - 7x$ $\qquad$ $y = 3x + 5$ $\qquad$ $y = 0$ $\qquad$ $y = 3x - 5$

$y = 7x - 5$

Intersect $\Rightarrow -5$: $y = 3x - 5$

(c)



Line A & Line D

$c = 3$

Which one of the five straight lines shown above could represent the equation $y = -2x + 3$?
Circle your answer. [1]

Line A $\qquad$ Line B $\qquad$ Line C $\qquad$ Line D $\qquad$ Line E

 (3300U50-1) **Turn over.**

Examiner only

10. A farmer knows that the time, t, taken by goats to eat all the grass in a particular field is inversely proportional to the number of goats, g, in the field.

When there are 25 goats in the field, the time taken to eat all the grass is 36 days.

You may assume that all the goats eat grass at the same rate.

(a) Find a formula for the time, t, in terms of the number of goats, g. [3]

$t \propto \frac{1}{g} \Rightarrow t = \frac{1}{g} \times k \Rightarrow t = \frac{k}{g}$

$t = k \times \frac{1}{g} \Rightarrow \frac{36}{1} \times \frac{k}{25}$

$t = \frac{k}{g}$ $\quad k = 36 \times 25 = \boxed{900}$

$\boxed{k = 900}$

$\boxed{t = \frac{900}{g}}$

(b) Hence, find the time taken for all of the grass to be eaten when there are 20 goats in the field. [1]

$t = \frac{k}{g} = \frac{\cancel{900}}{\cancel{20}} = 45$

$t = 45$ (days)

(c) The farmer needs the grass to last for at least 40 days.
What is the greatest number of goats that should be allowed in the field? [2]

$t = 40$ days ; $g = ?$

Recall: $t = \frac{k}{g} \Rightarrow k = 900$

$\frac{40}{1} \times \frac{900}{g} \Rightarrow \frac{\cancel{40}g}{\cancel{40}} = \frac{\overset{45}{\cancel{900}}}{\underset{2}{\cancel{40}}}$

$g = \frac{45}{2} = 22.5 = \boxed{22 \text{ goats}}$

14

Examiner only

11. *(a)* Circle the expression which is equivalent to $m^{\frac{2}{3}}$. [1]

$\frac{1}{3}m^2$ $\qquad 2m^{\frac{1}{3}}$ $\qquad \frac{2}{3}m$ $\qquad \left(\sqrt[3]{m}\right)^2$ $\qquad \left(\sqrt{m}\right)^3$

$m^{2/3} = \sqrt[3]{m^2} = \left(\sqrt[3]{m}\right)^2$

$= m^{\frac{1}{3}\times 2} \Rightarrow \left(m^{\frac{1}{3}}\right)^2 \Rightarrow \left(\sqrt[3]{m}\right)^2$

(b) Circle the expression which is equivalent to $p^{\frac{3}{4}} \times p^{-\frac{1}{4}} \div p^{\frac{1}{4}}$. [1]

$p^{-\frac{1}{4}}$ $\qquad p^{-\frac{3}{64}}$ $\qquad p^{\frac{5}{4}}$ $\qquad p^{\frac{3}{4}}$ $\qquad p^{\frac{1}{4}}$

$\Rightarrow p^{\frac{3}{4}} \times p^{-\frac{1}{4}} \div p^{\frac{1}{4}} \Rightarrow p^{\frac{3}{4}+-\frac{1}{4}-\frac{1}{4}} \Rightarrow p^{\frac{3}{4}-\frac{1}{4}-\frac{1}{4}}$

$\Rightarrow p^{\frac{3-1-1}{4}} = (p)^{\frac{1}{4}} \Rightarrow p^{1/4}$

12. Express the following as a single fraction in its simplest form. [3]

$$\frac{6}{3x-5} - \frac{4}{2x+1}$$

$\frac{6}{3x-5} - \frac{4}{2x+1}$

$\Rightarrow \frac{6(2x+1) - 4(3x-5)}{(3x-5)(2x+1)}$

$\Rightarrow \frac{12x+6-12x+20}{(3x-5)(2x+1)} = \frac{\cancel{12x}-\cancel{12x}+6+20}{(3x-5)(2x+1)}$

$\Rightarrow \frac{26}{(3x-5)(2x+1)}$

 (3300U50-1) **Turn over.**

Examiner only

13. Two similar cones have volumes of 20 cm^3 and 1280 cm^3.
The radius of the base of the smaller cone is 2·3 cm.
Calculate the radius of the base of the larger cone. [3]

$4 \times 4 \times 4$

$V_1 = 20 cm^3$; $V_2 = 1280 cm^3$

Since V_1 & V_2 are two Similar Cones

r = radius of the base of the Smaller Cone

$r = 2.3 cm$

R = radius of the base of the Larger Cone

$R = \sqrt[3]{\frac{\cancel{1280}\ V_2}{\cancel{20}\ V_1}} \times r$

$= \sqrt[3]{64} \times 2.3 \Rightarrow 4 \times 2.3 = 9.2$

$R = \underline{9.2 cm}$

$23 \times 4 = 92$

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14. *(a)* Express $0{\cdot}8\dot{1}\dot{2}$ as a fraction. $\Rightarrow \boxed{\frac{134}{165}}$ [2]

0.8121212 ......

$\Rightarrow$ Let $\frac{x}{1} \times \frac{8.121212\ldots}{10}$; Let $x = \frac{812.1212\ldots}{1000}$

$10x = 8.121212\ldots$; $1000x = 812.121212\ldots$

$\Rightarrow$ Subtracting $10x$ from $1000x$

$1000x - 10x = (812.\cancel{121212}\ldots) - (8.\cancel{121212}\ldots)$

$\frac{990x}{990} = \frac{804}{990}$ $\Rightarrow x = \frac{\cancel{402}}{\cancel{495}} = \frac{134}{165}$

495

134
6

812.1212
8.1212
804.0000

(b) Simplify $\sqrt{72}$.
Circle your answer. [1]

$2\sqrt{6}$ $\quad$ $6\sqrt{2}$ $\quad$ $6\sqrt{12}$ $\quad$ 36 $\quad$ $36\sqrt{2}$

$\Rightarrow \sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2}$

$= 6 \times \sqrt{2} = 6\sqrt{2}$

(c) Expand and simplify $(7-2\sqrt{5})(3+\sqrt{5})$. [2]

$(7-2\sqrt{5})(3+\sqrt{5})$

$\Rightarrow 7(3+\sqrt{5}) - 2\sqrt{5}(3+\sqrt{5})$

$\Rightarrow 21 + 7\sqrt{5} - 6\sqrt{5} - 2\sqrt{25}$

$\Rightarrow 21 + 7\sqrt{5} - 6\sqrt{5} - 2(5)$

$\Rightarrow 21 + \sqrt{5} - 10$

$\Rightarrow 21 - 10 + \sqrt{5}$

$\Rightarrow 11 + \sqrt{5}$

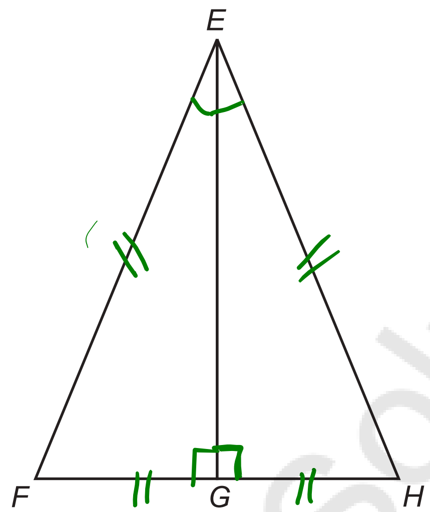
1
4
9
16
25
36
49
64

 (3300U50-1) **Turn over.**

Examiner only

15. In the triangle *EFH* below:

- *G* is the midpoint of *FH*,
- *EG* and *FH* are perpendicular.



SSA

Prove that *EFG* and *EHG* are congruent triangles.
You must state the condition of congruence. [4]

FG = HG ⇒ S

EG = Common side for △ EFG & △ EGH = S

Angle EGF = Angle EGH = 90° = A

SAS (2 Sides & an Included Angle)

∴ EFG & EHG are Congruent triangles

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Examiner only

16. Make y the subject of the following formula. [4]

$$2y = \sqrt{3 + my^2}$$

$2y = \sqrt{3+my^2}$

Take the square of both sides

$\Rightarrow (2y)^2 = (\sqrt{3+my^2})^2$

$(2y)^2 = 3+my^2$

$2y \times 2y = 3+my^2$

$4y^2 = 3+my^2$

$4y^2 - my^2 = 3$

$\frac{y^2(\cancel{4-m})}{\cancel{4-m}} = \frac{3}{4-m} \Rightarrow y^2 = \frac{3}{4-m}$

Square root of both sides

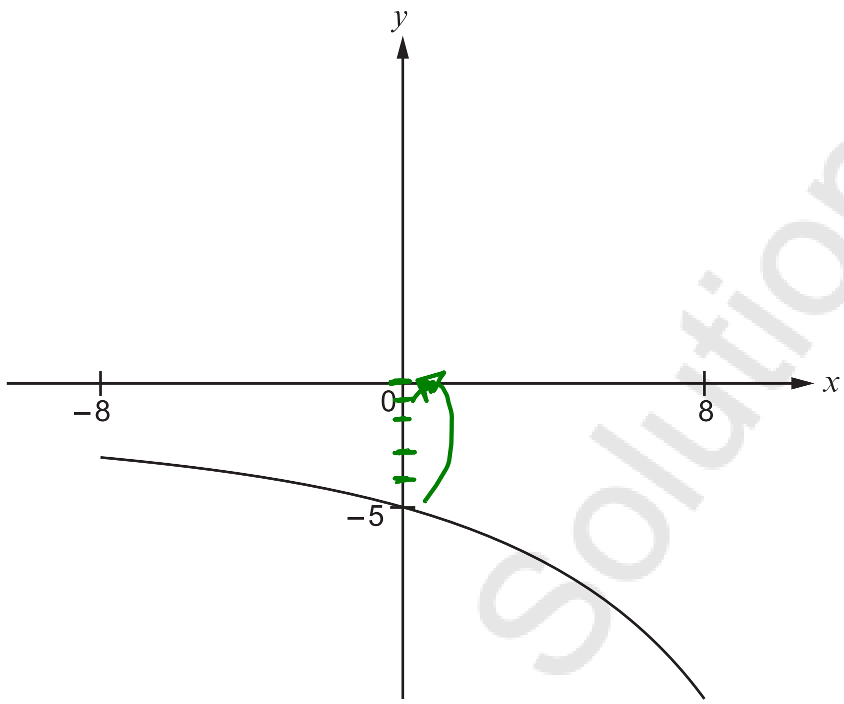
$\cancel{\pm\sqrt{y^2}} = \pm\sqrt{\frac{3}{4-m}}$

$y^2 = \pm\sqrt{\frac{3}{4-m}}$

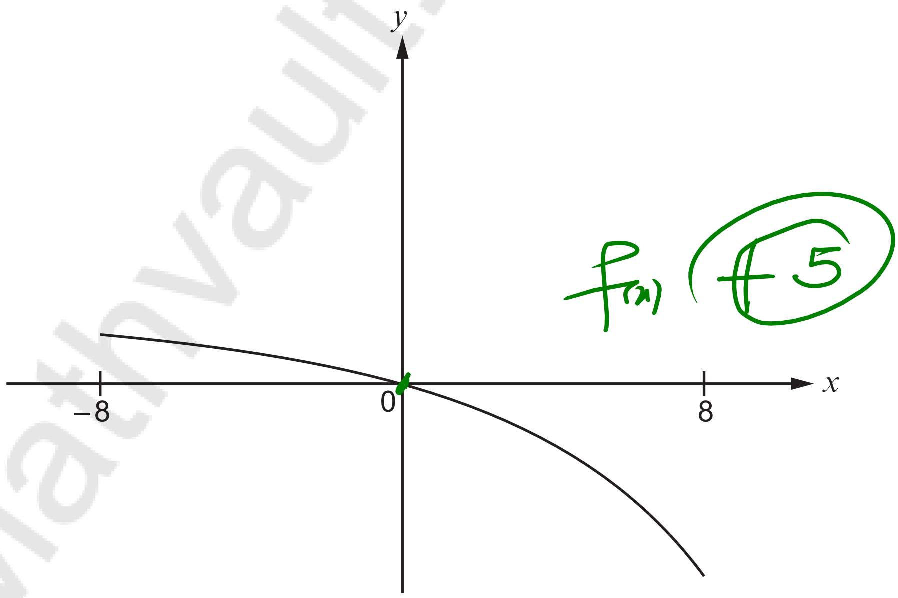
 (3300U50-1) **Turn over.**

Examiner only

17. *(a)* The following diagram shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.



Using function notation, complete the equation of the transformed curve. [1]

$y =$ f(x) + 5

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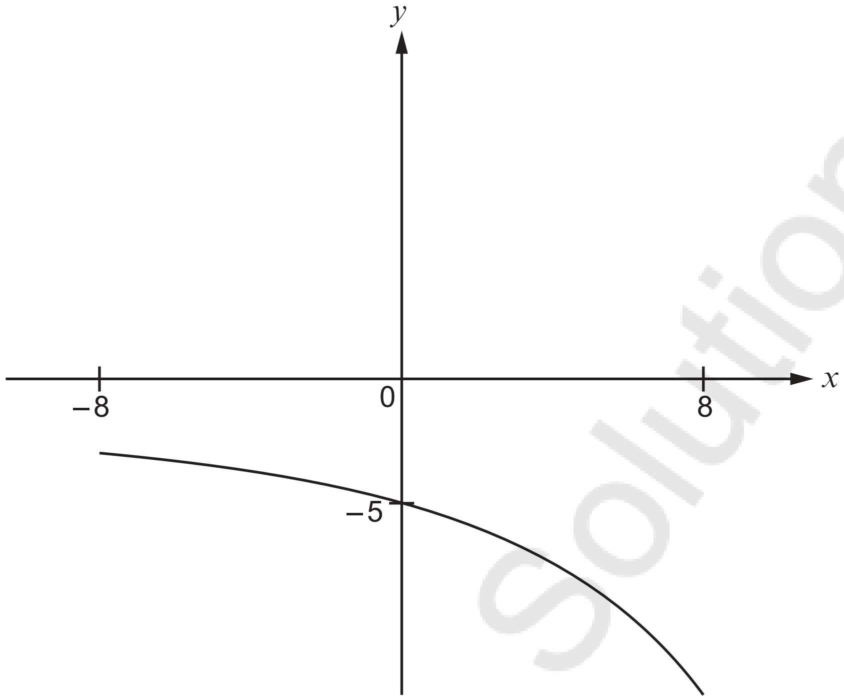
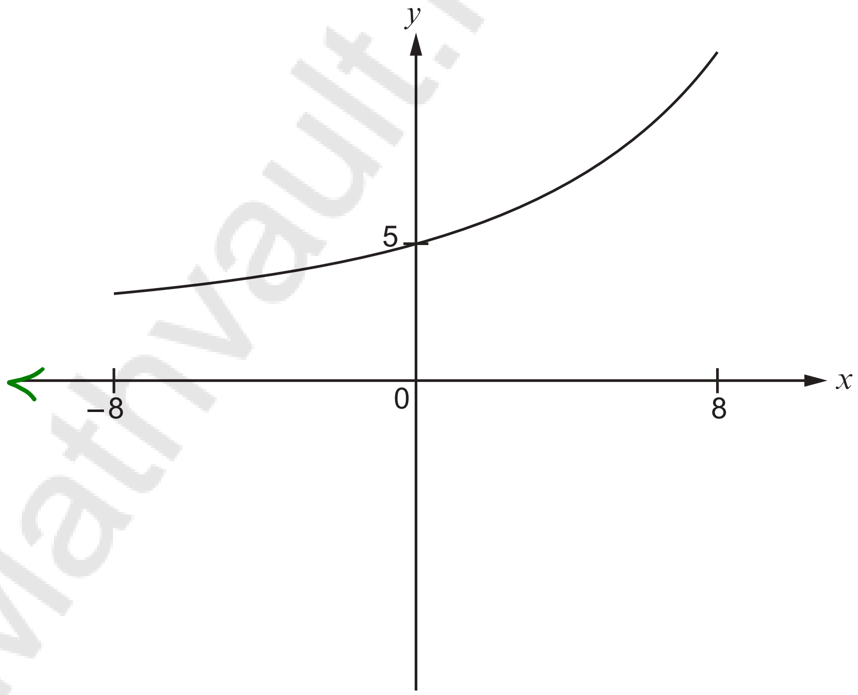
(b) The following diagram again shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.



Using function notation, complete the equation of the transformed curve. [1]

$y =$ ..... $-f(x)$ .....

Turn over.

Examiner only

18. A circle has radius r cm, where r is an integer.
The side of a square is of length x cm.

If the circle and square have the same area, explain why x cannot be an integer.

You should consider algebraic expressions in your answer. [2]

Area of a Circle $= \pi r^2$

Sign of length x cm) $= \sqrt{(A_c)}$

$= \sqrt{\pi r^2} = \sqrt{\pi} \times \sqrt{r^2} = \sqrt{\pi} \times r$

Length x is not an integer because of $\sqrt{\pi}$ where $\sqrt{\pi}$ is an irrational no

$\therefore$ x is Irrational

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Examiner only

19. Dewi has a box containing eleven socks.
Six of the socks are red, four are green and one is yellow.

Early one morning, without switching on the light, Dewi selects two socks at random.

(a) Calculate the probability that the first sock selected is yellow and the second is red. [2]

$P(YR) = P(Y) \text{ and } P(R)$

$= \frac{1}{11} \times \frac{6}{10} = \cancel{\frac{6}{110}} = \frac{3}{55}$

(b) Calculate the probability that Dewi selects two socks of the same colour. [3]

$P(\text{two sock of the same colour}) = P(YY) \text{ or } P(RR) \text{ or } P(GG)$

$= \left(\frac{1}{11} \times \frac{0}{11}\right) + \left(\frac{\cancel{6}^{3}}{11} \times \frac{\cancel{5}^{1}}{\cancel{10}_{2}}\right) + \left(\frac{\cancel{4}^{2}}{11} \times \frac{3}{\cancel{10}_{5}}\right)$

$= 0 + \frac{3}{11} + \frac{6}{55} = \frac{15+6}{55} = \frac{21}{55}$

(c) Calculate the probability that at least one green sock is selected. [3]

$P(\text{at least one Green Sock}) = P(\square\,\square)$

$= [P(GR) \text{ or } P(GG) \text{ or } P(GY)] \text{ or } P(RG) \text{ or } P(YG)$

$= \left(\frac{\cancel{4}^{2}}{11} \times \frac{6}{\cancel{10}_{5}}\right) + \left(\frac{\cancel{4}^{2}}{11} \times \frac{3}{\cancel{10}_{5}}\right) + \left(\frac{\cancel{4}^{2}}{11} \times \frac{1}{\cancel{10}_{5}}\right) + \left(\frac{\cancel{6}^{3}}{11} \times \frac{4}{\cancel{10}_{5}}\right) + \left(\frac{1}{11} \times \frac{\cancel{4}^{2}}{\cancel{10}_{5}}\right)$

$= \frac{12}{55} + \frac{6}{55} + \frac{2}{55} + \frac{12}{55} + \frac{2}{55}$

$= \frac{12+6+2+12+2}{55} = \frac{34}{55}$

END OF PAPER

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Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

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